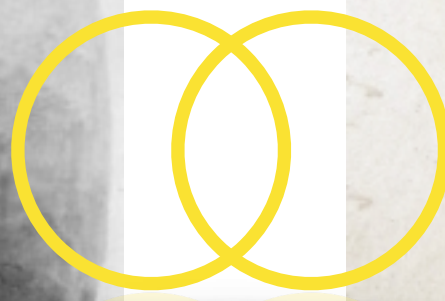


# A journey of (in-)commensuration

from  
pythagorean harmony  
to  
modern quantum physics

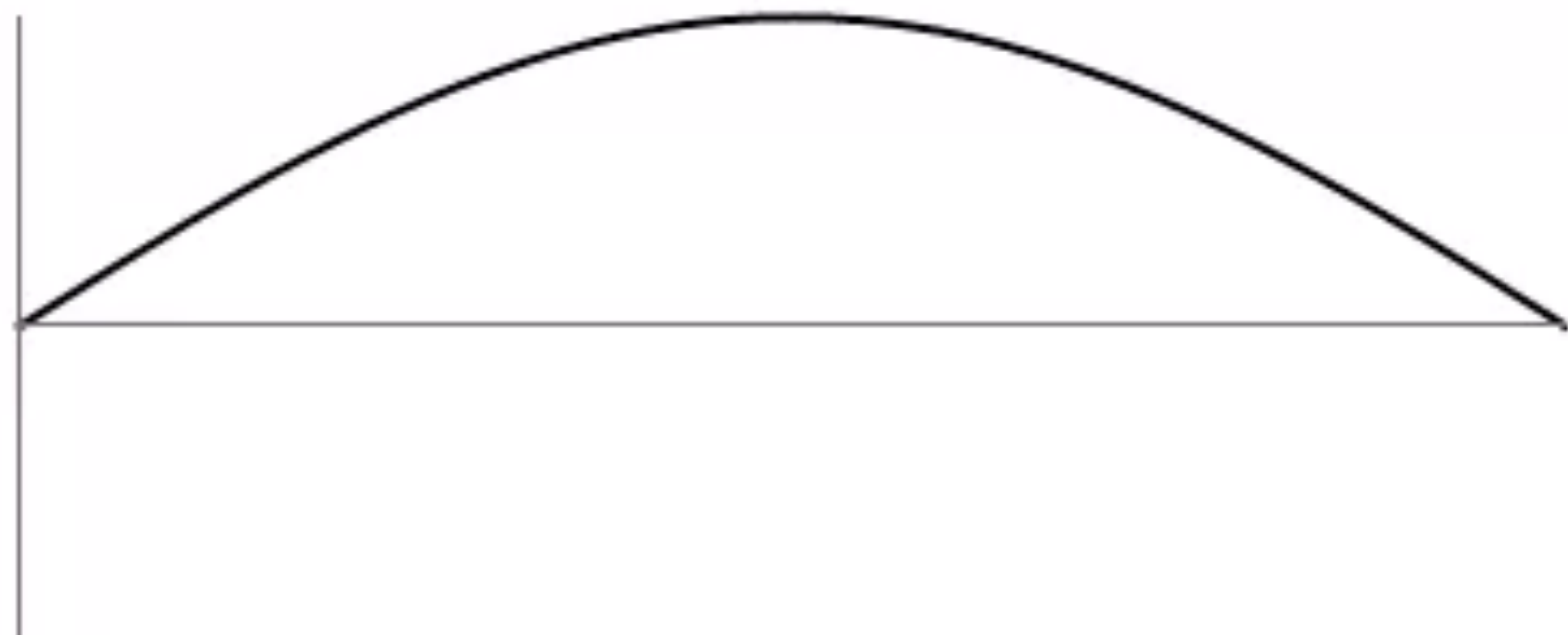
Elio König-Tarasevich

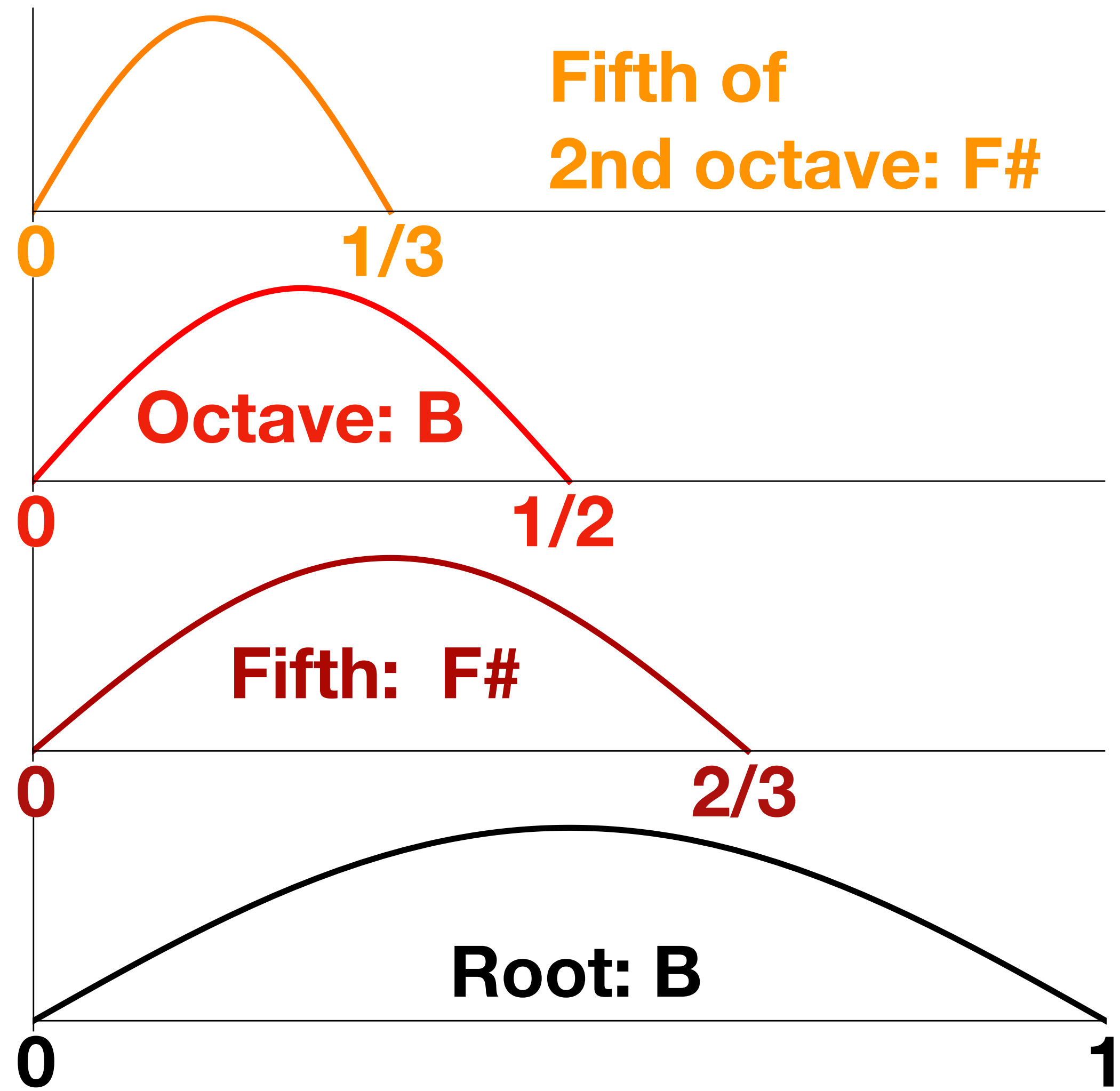
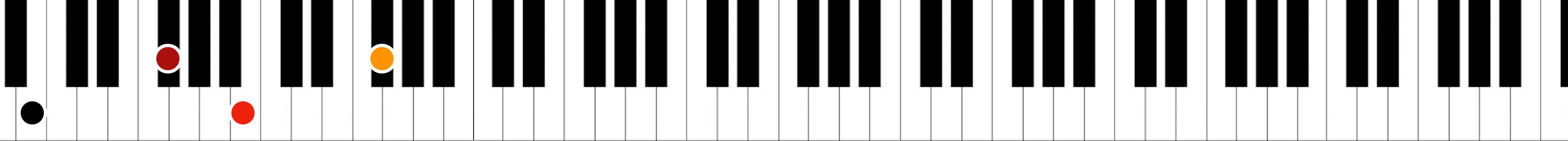




**J.P.G. Lejeune  
Dirichlet  
(1805-1859)**

**R. Mendelssohn  
Bartholdy  
(1811-1858)**

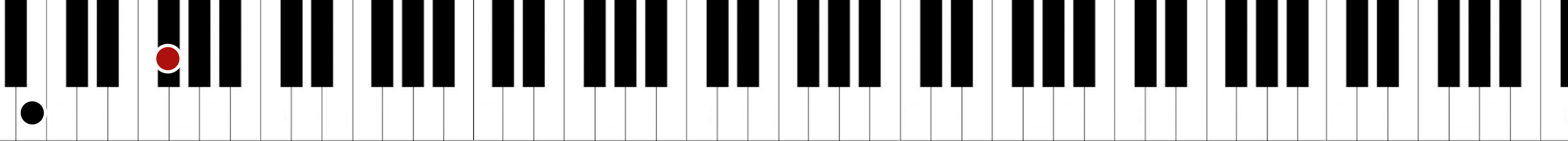




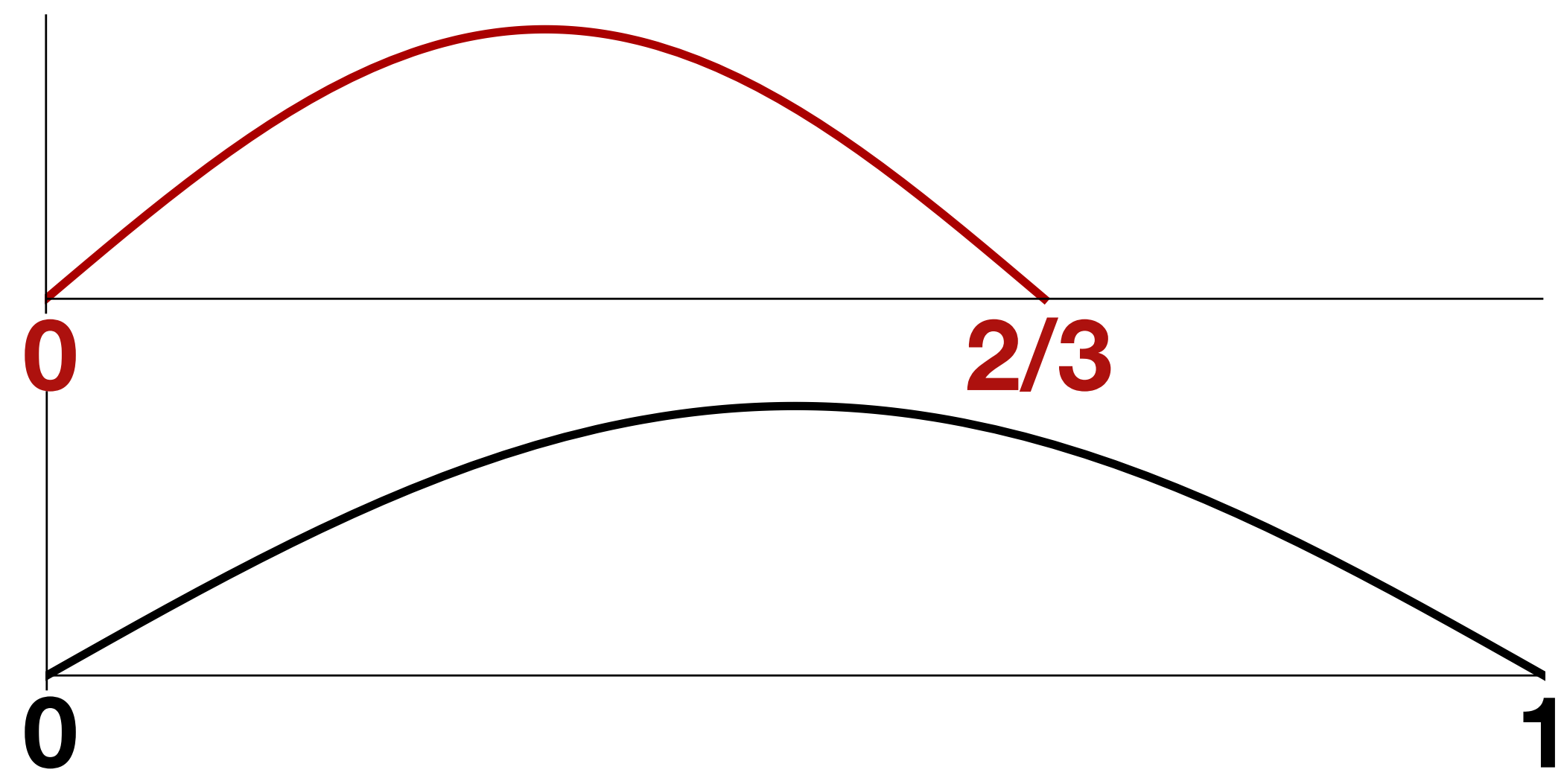
**2nd overtone**

**1st overtone**

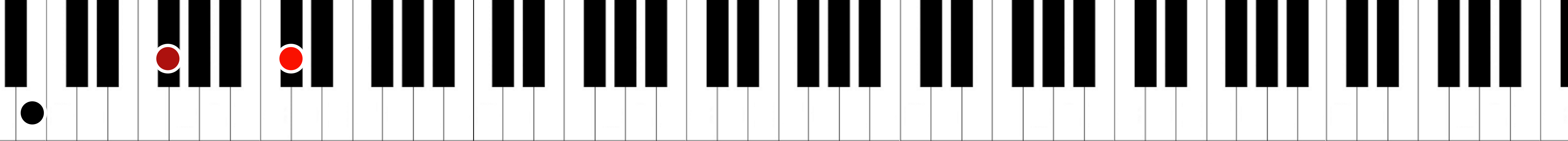
**Fundamental tone**



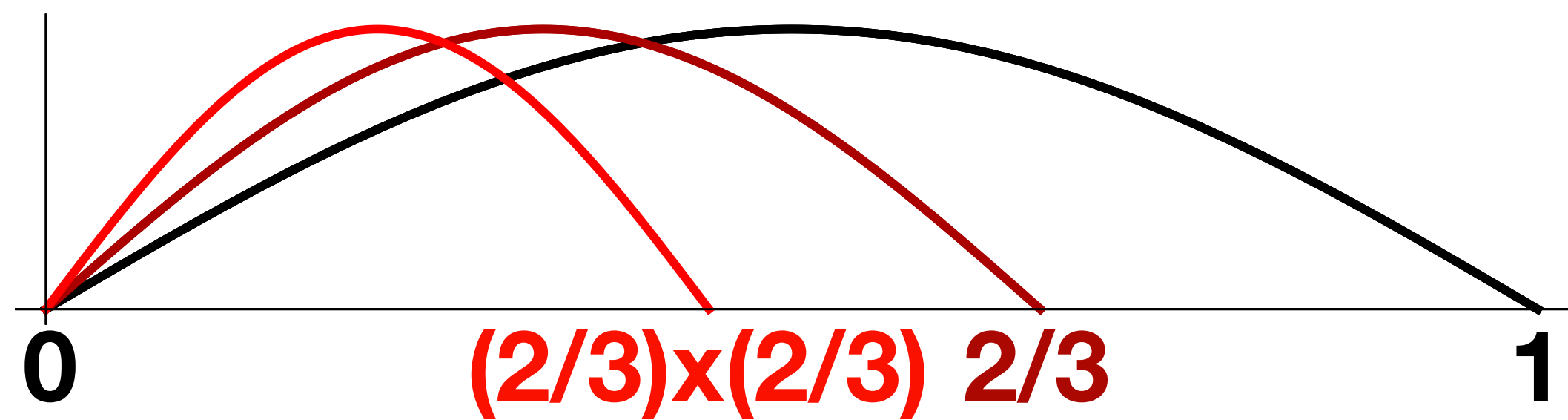
# Tuning following the cycle of fifths



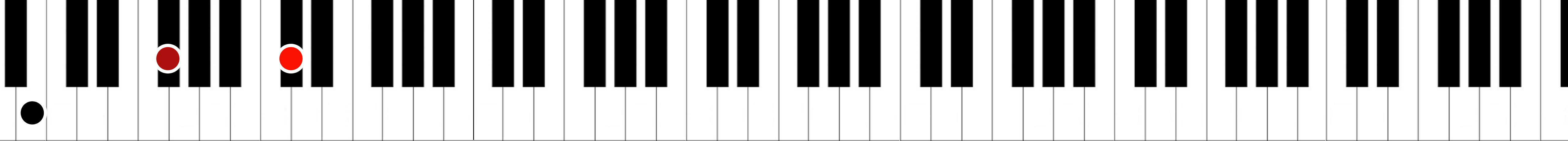




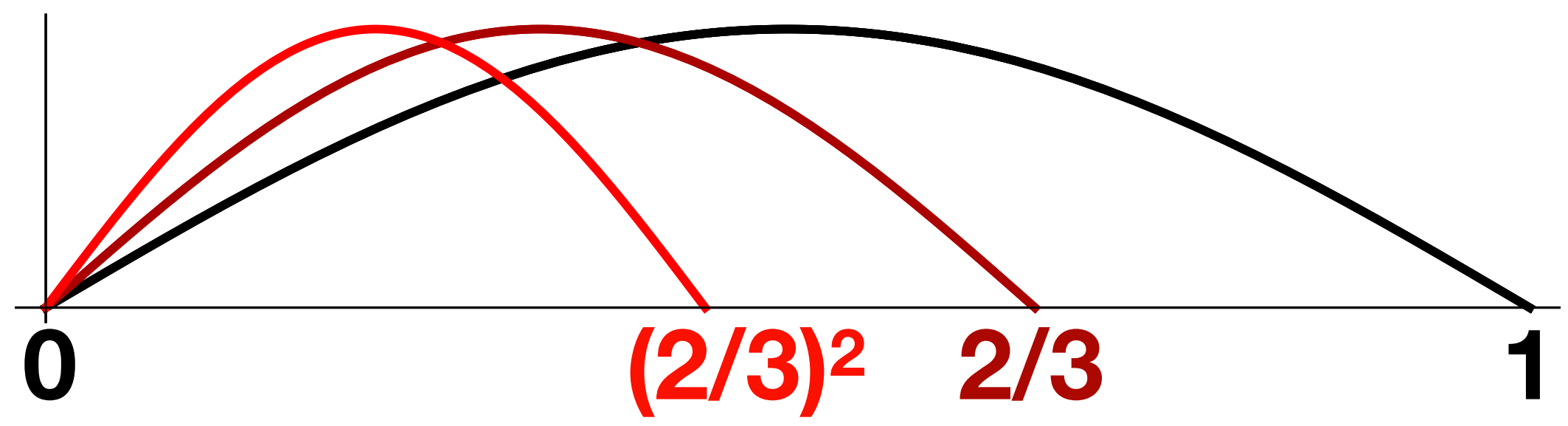
# Tuning following the cycle of fifths



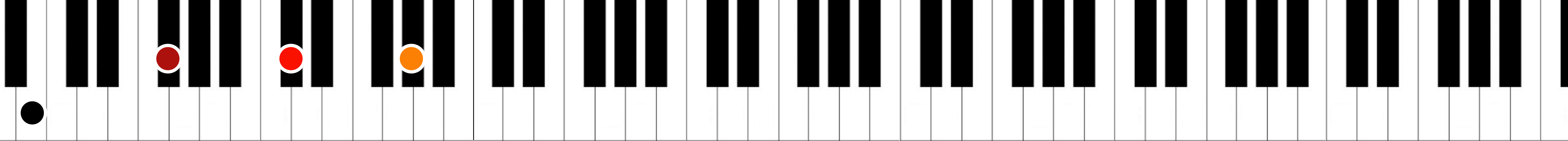




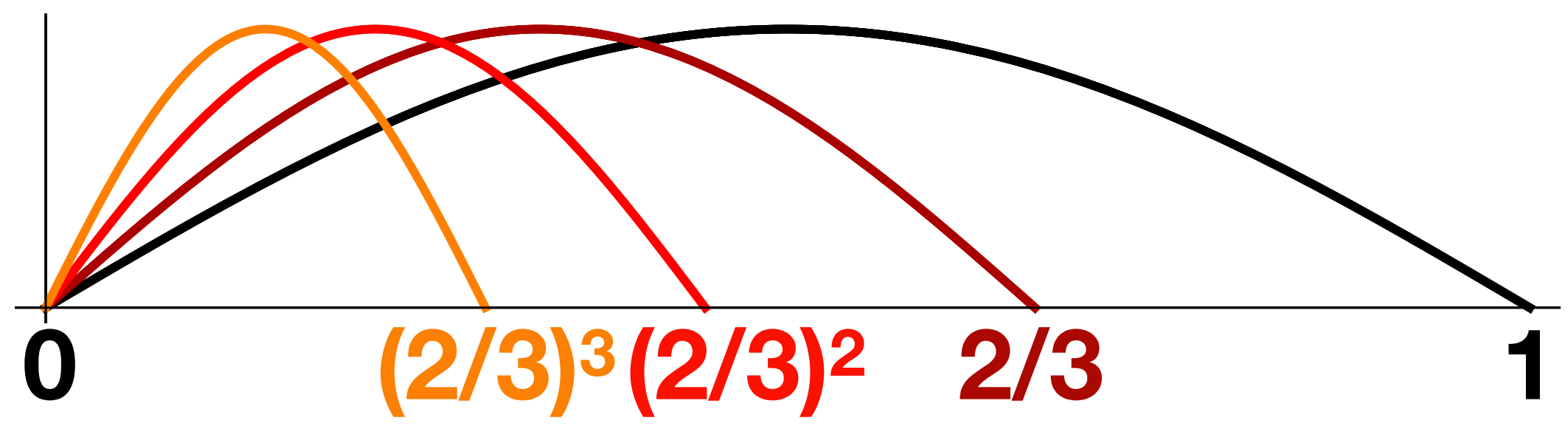
# Tuning following the cycle of fifths



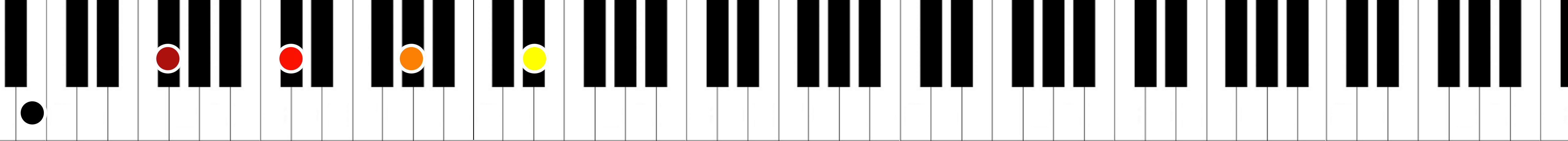




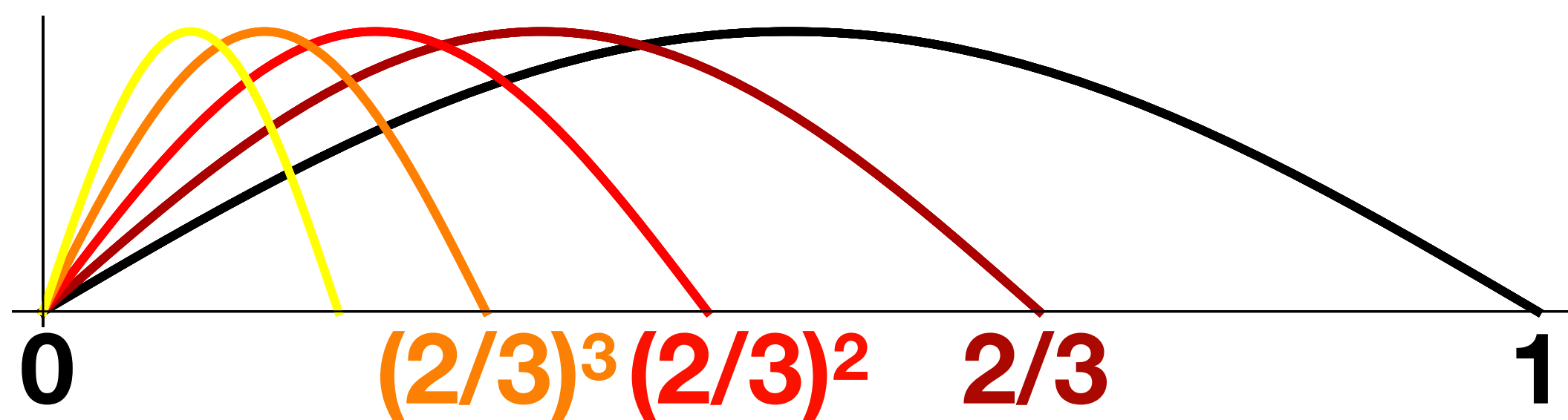
# Tuning following the cycle of fifths

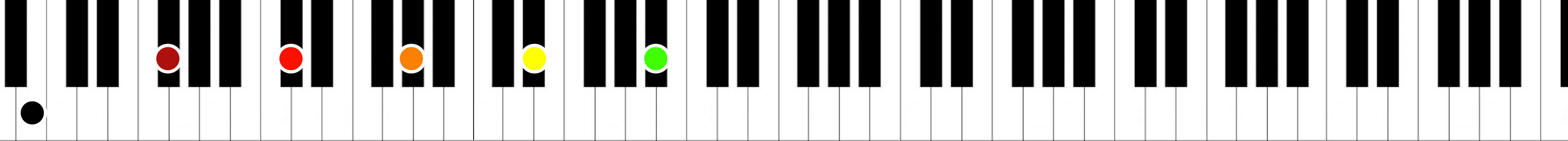




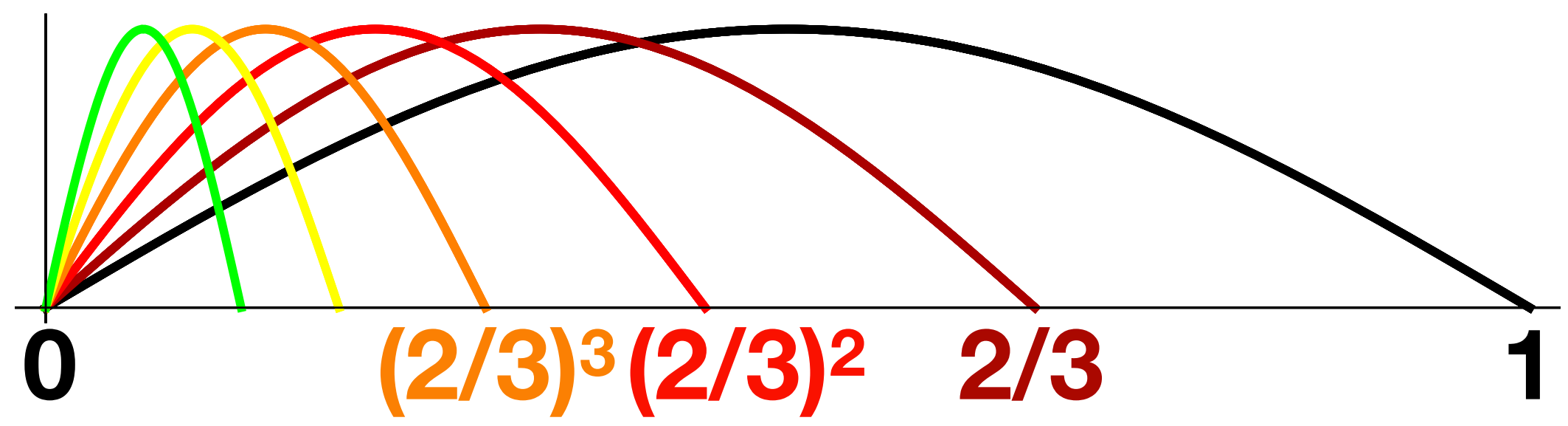


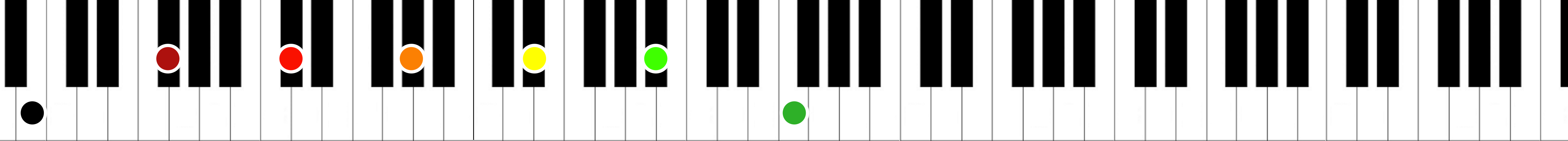
# Tuning following the cycle of fifths



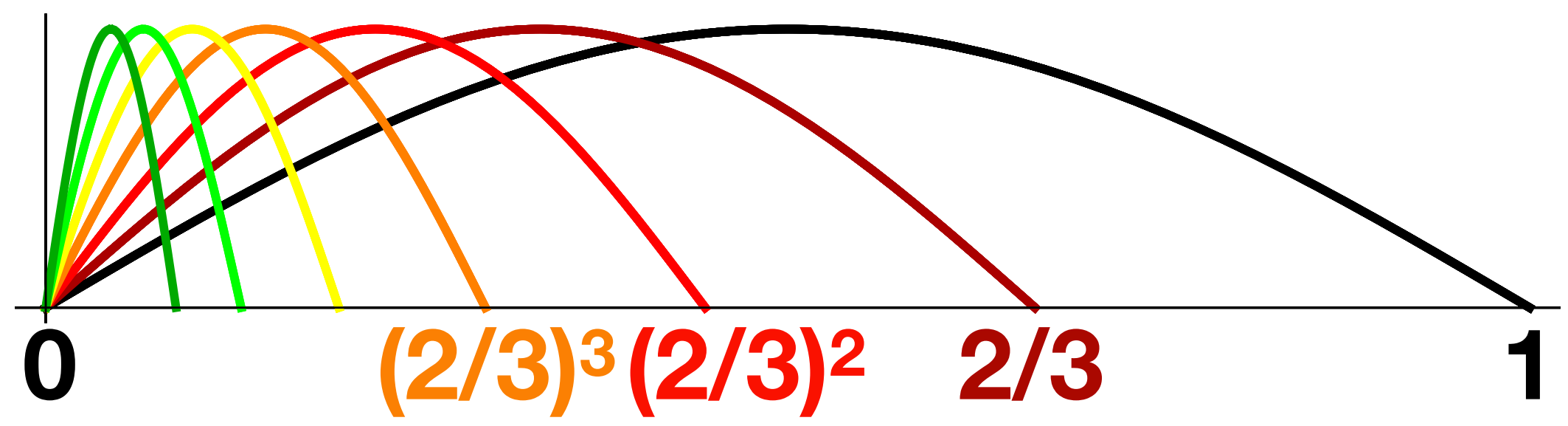


# Tuning following the cycle of fifths

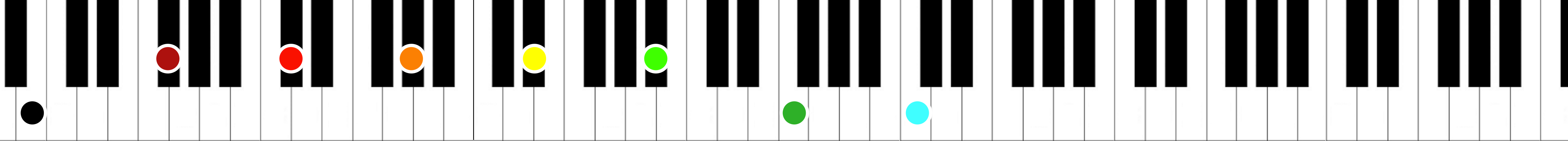




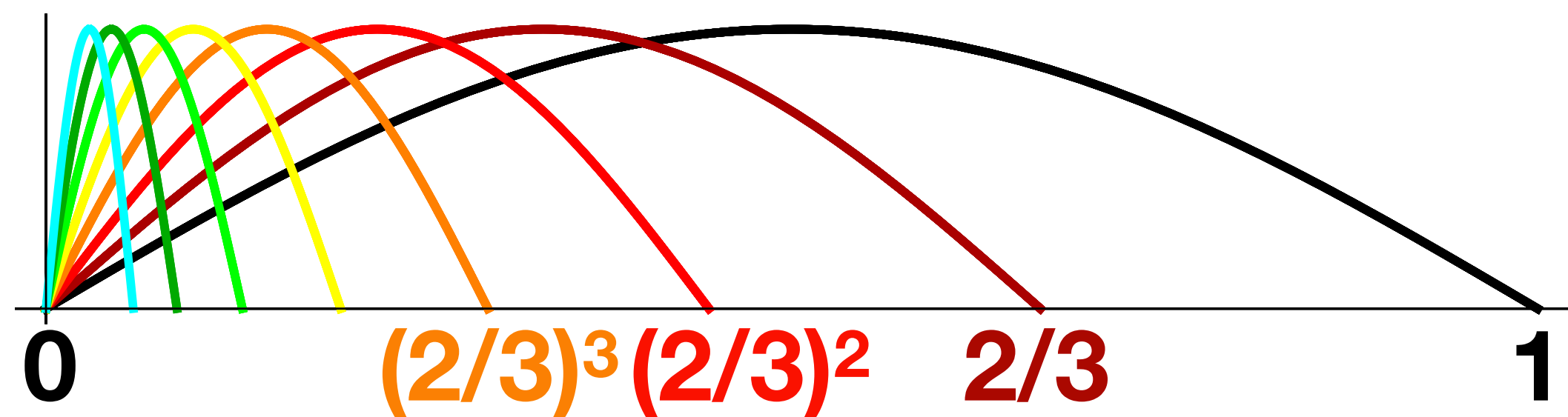
# Tuning following the cycle of fifths

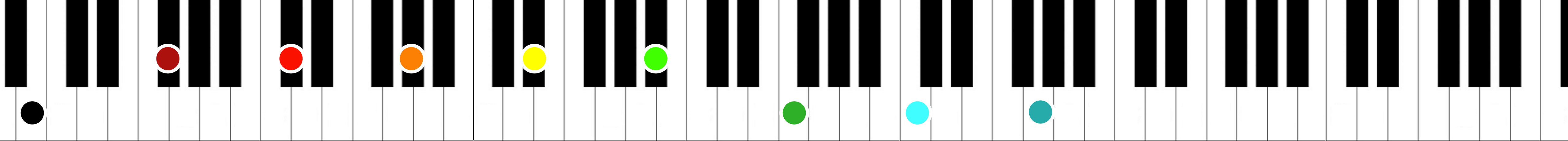




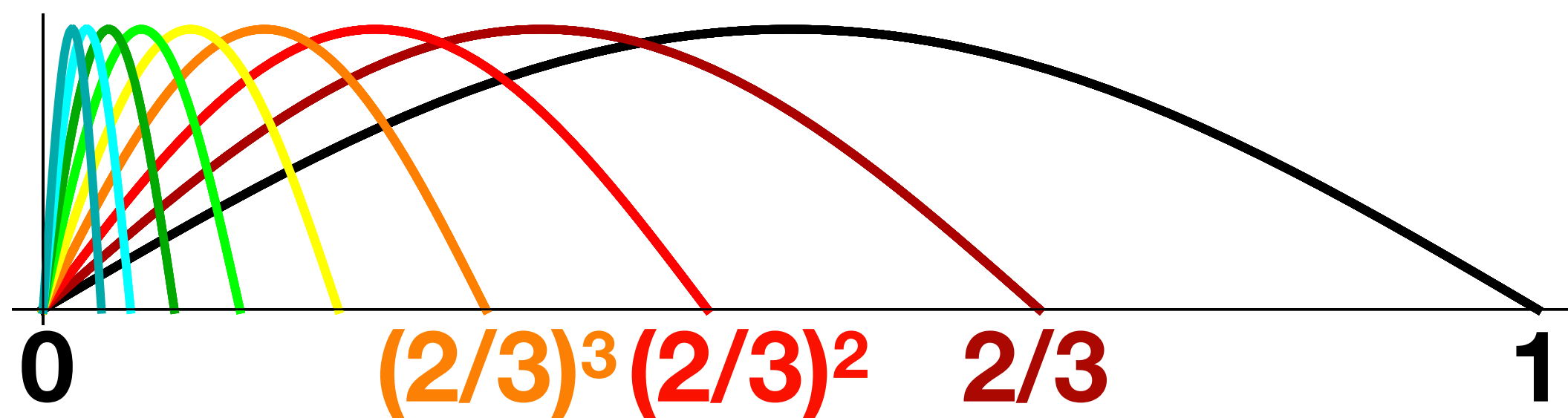


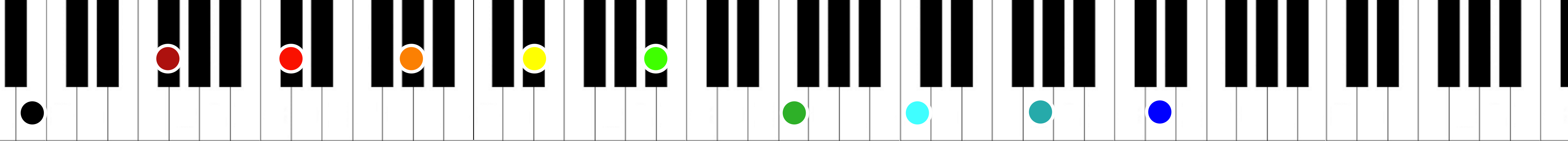
# Tuning following the cycle of fifths



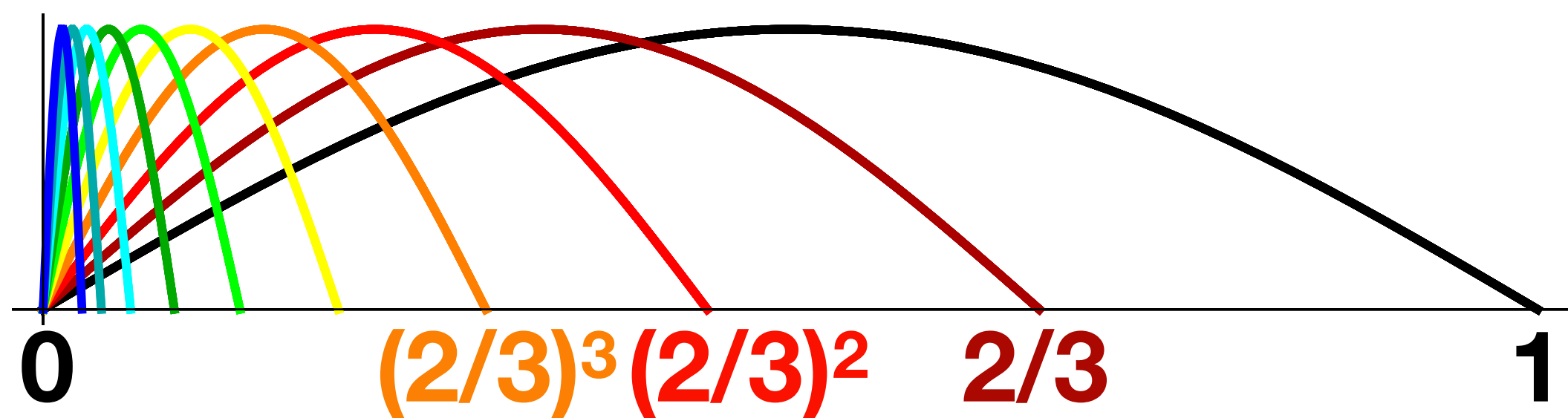


# Tuning following the cycle of fifths

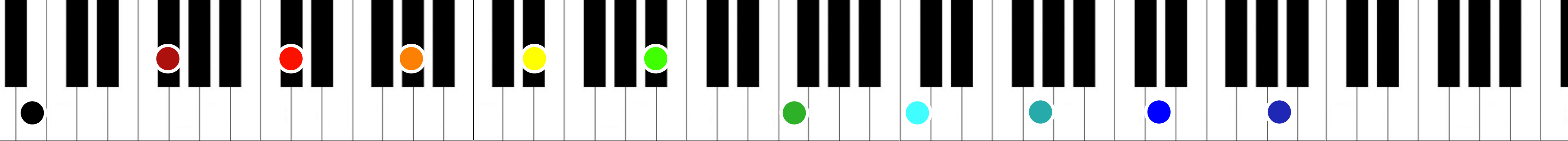




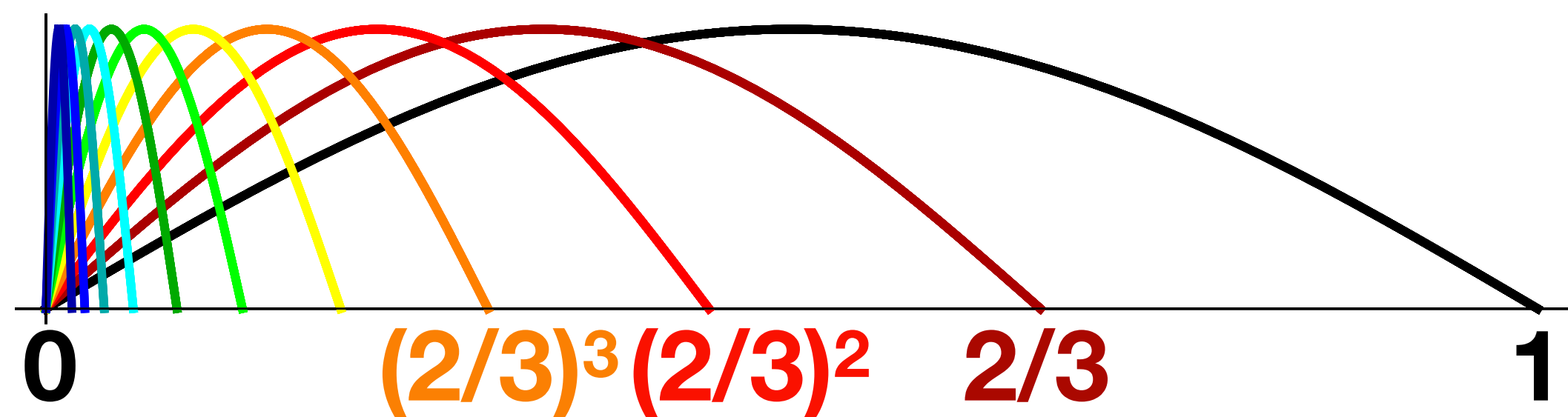
# Tuning following the cycle of fifths

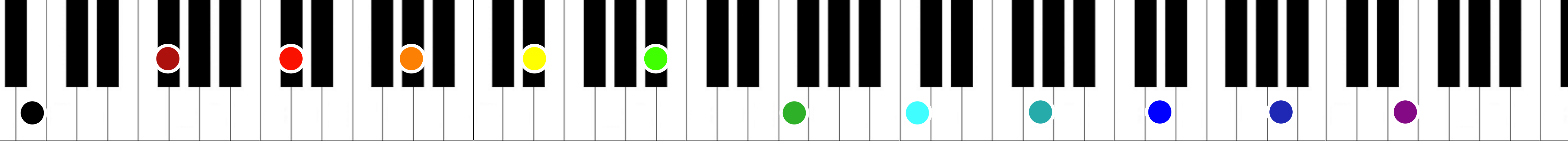




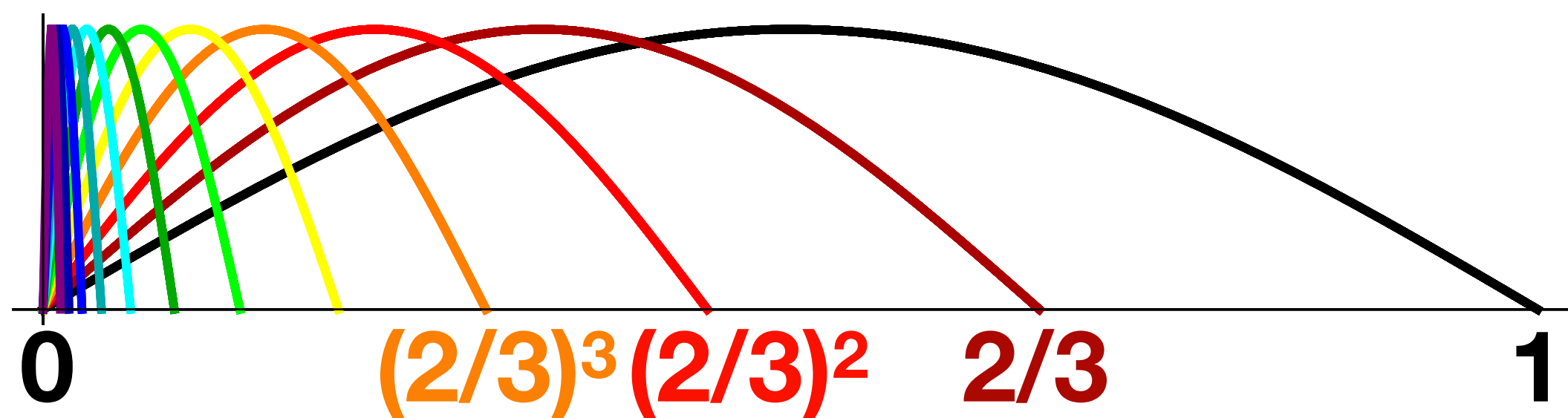


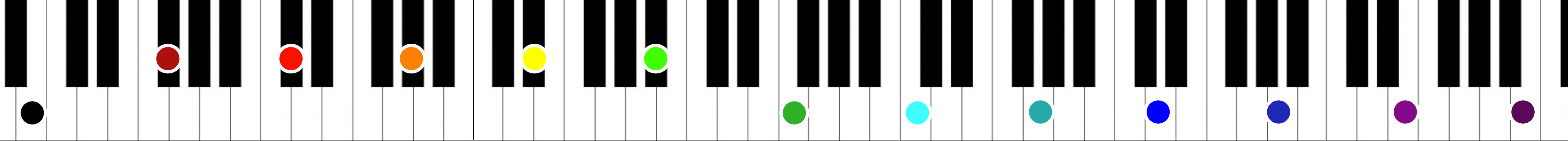
# Tuning following the cycle of fifths



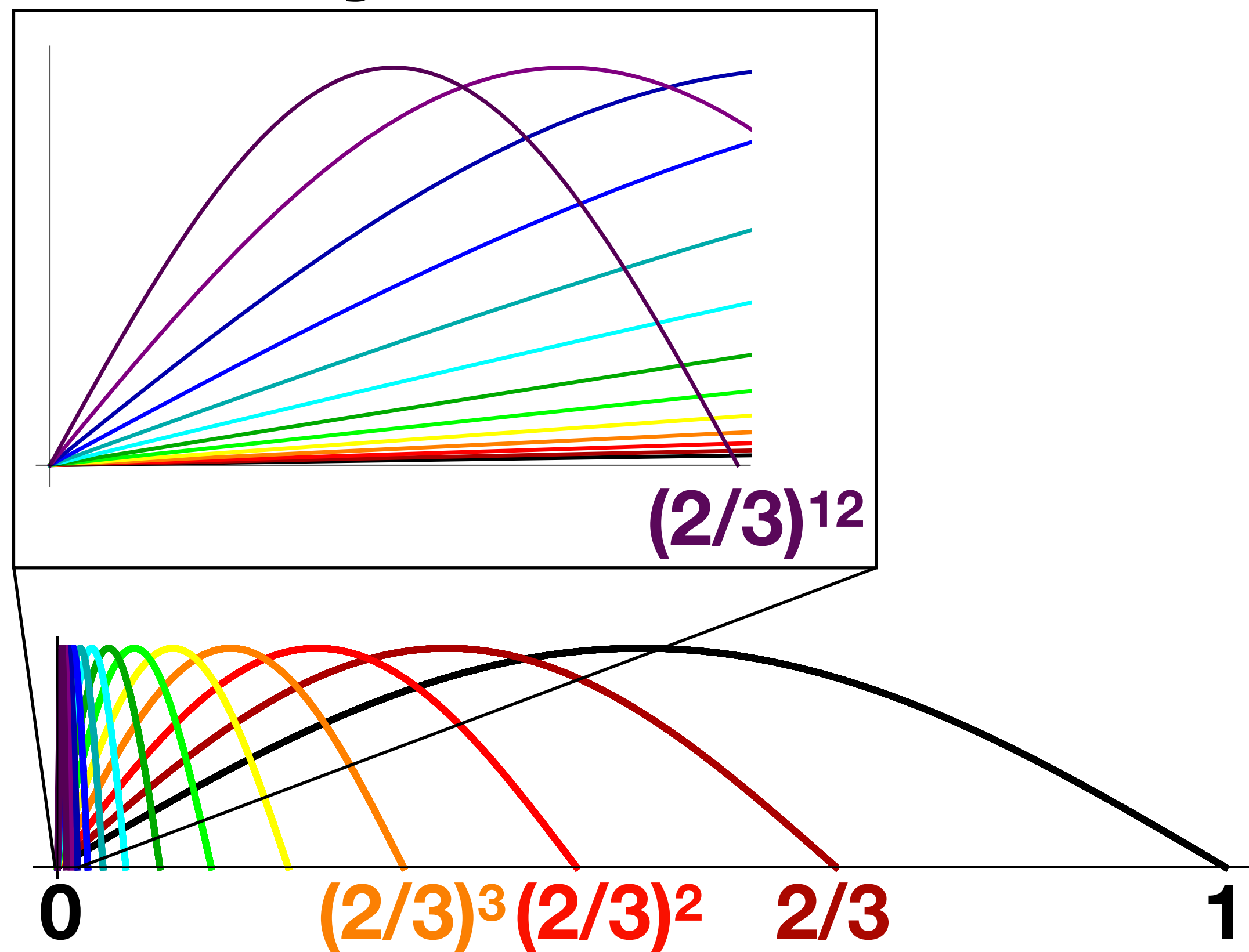


# Tuning following the cycle of fifths

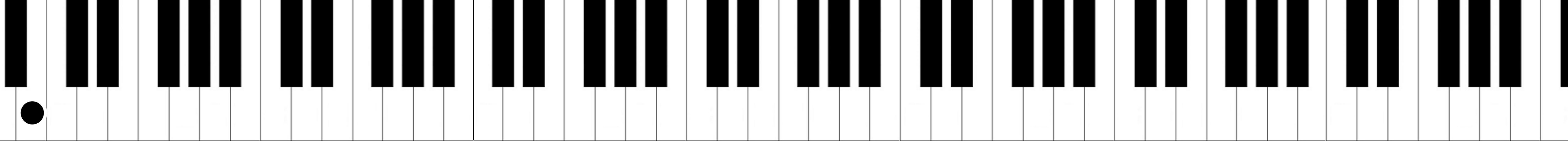




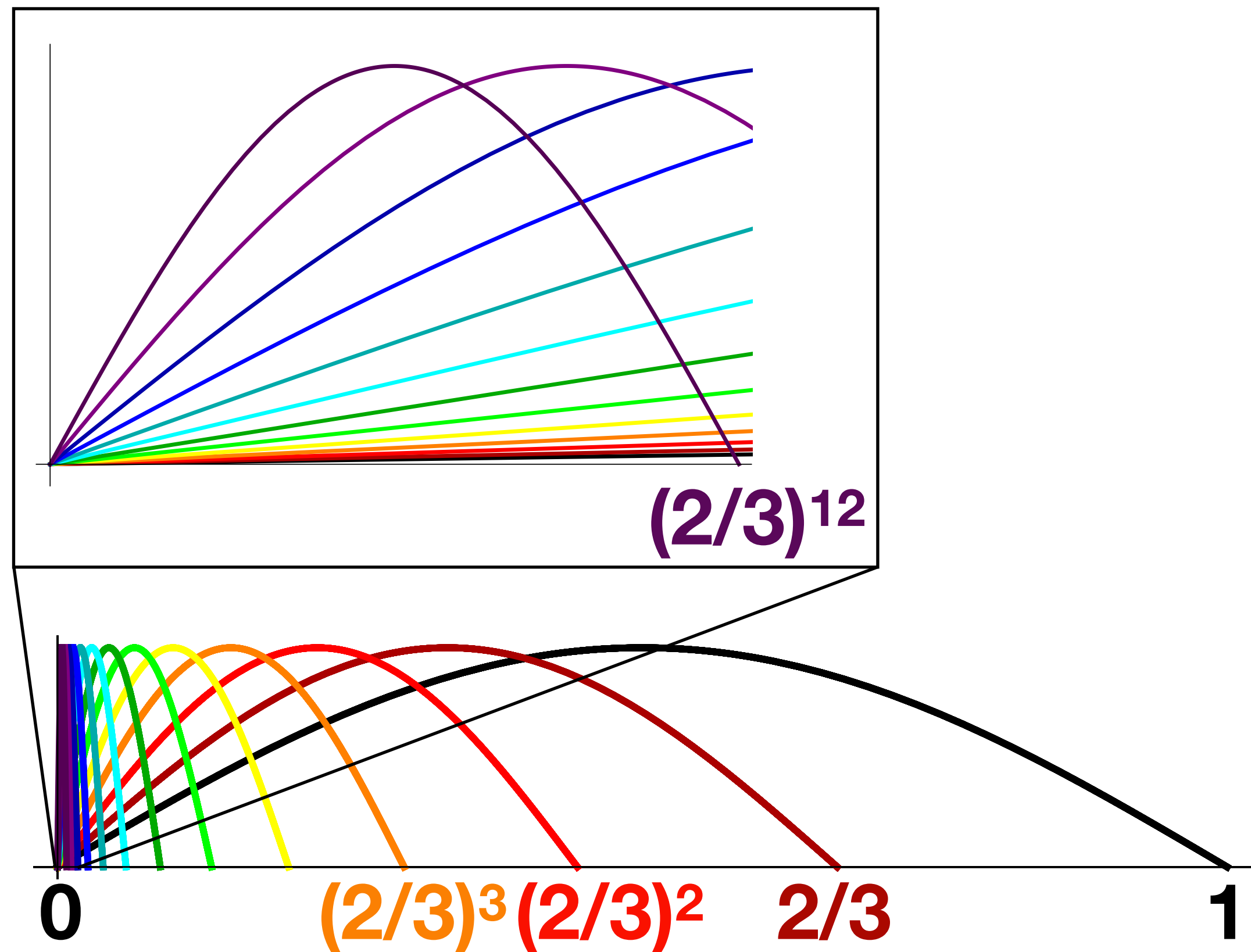
# Tuning following the cycle of fifths



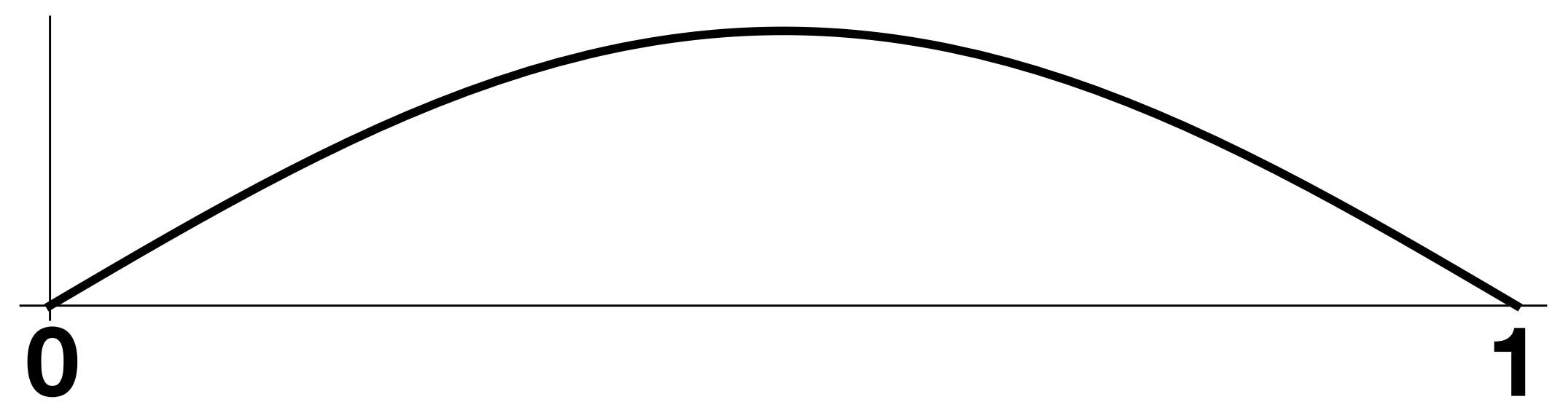


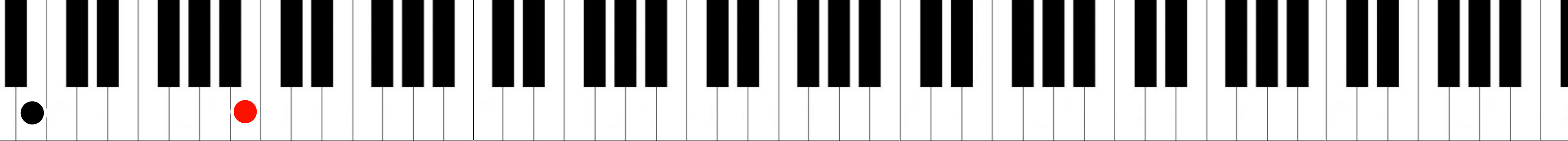


## Tuning following the cycle of fifths

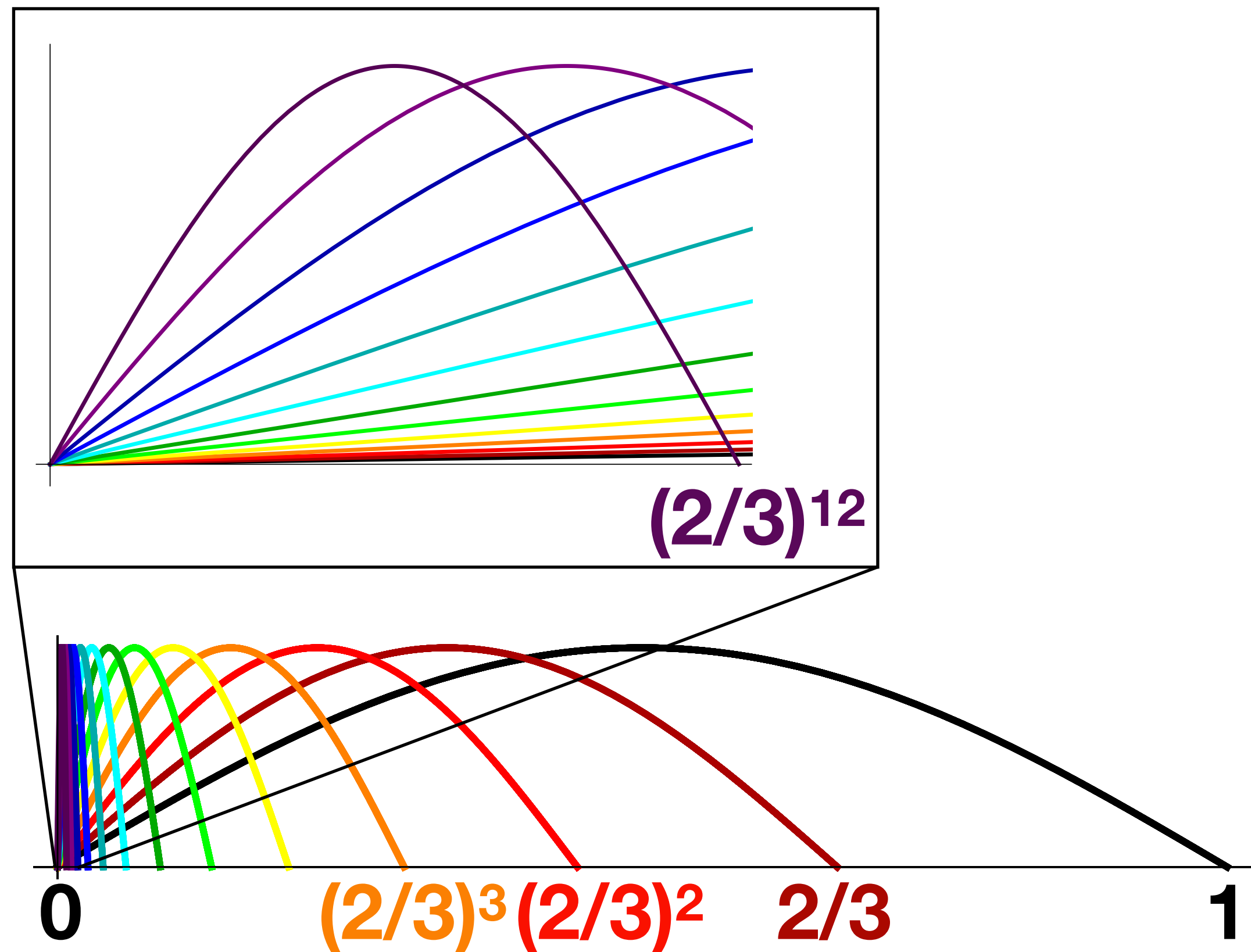


## Comparison to “cycle” of octaves

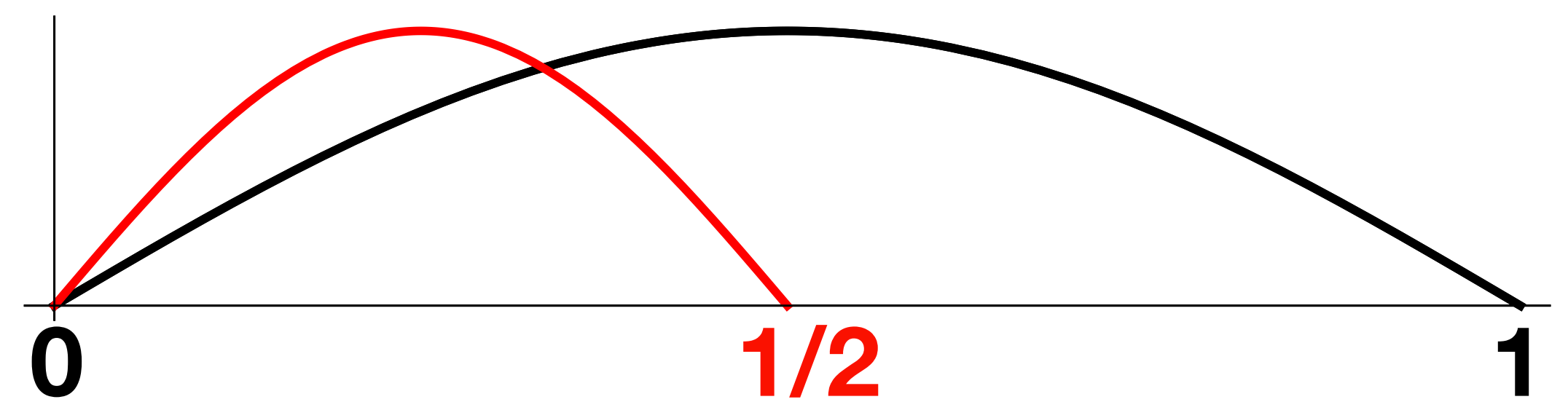


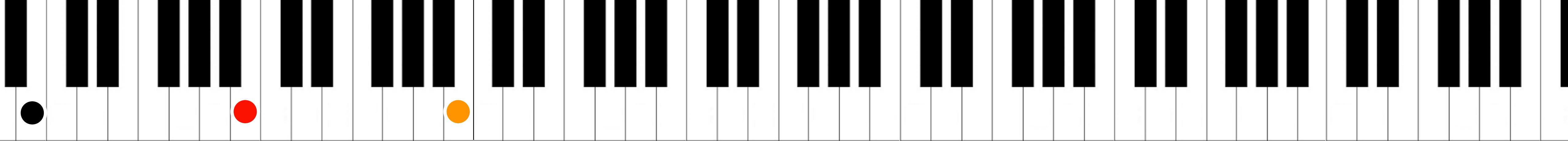


# Tuning following the cycle of fifths

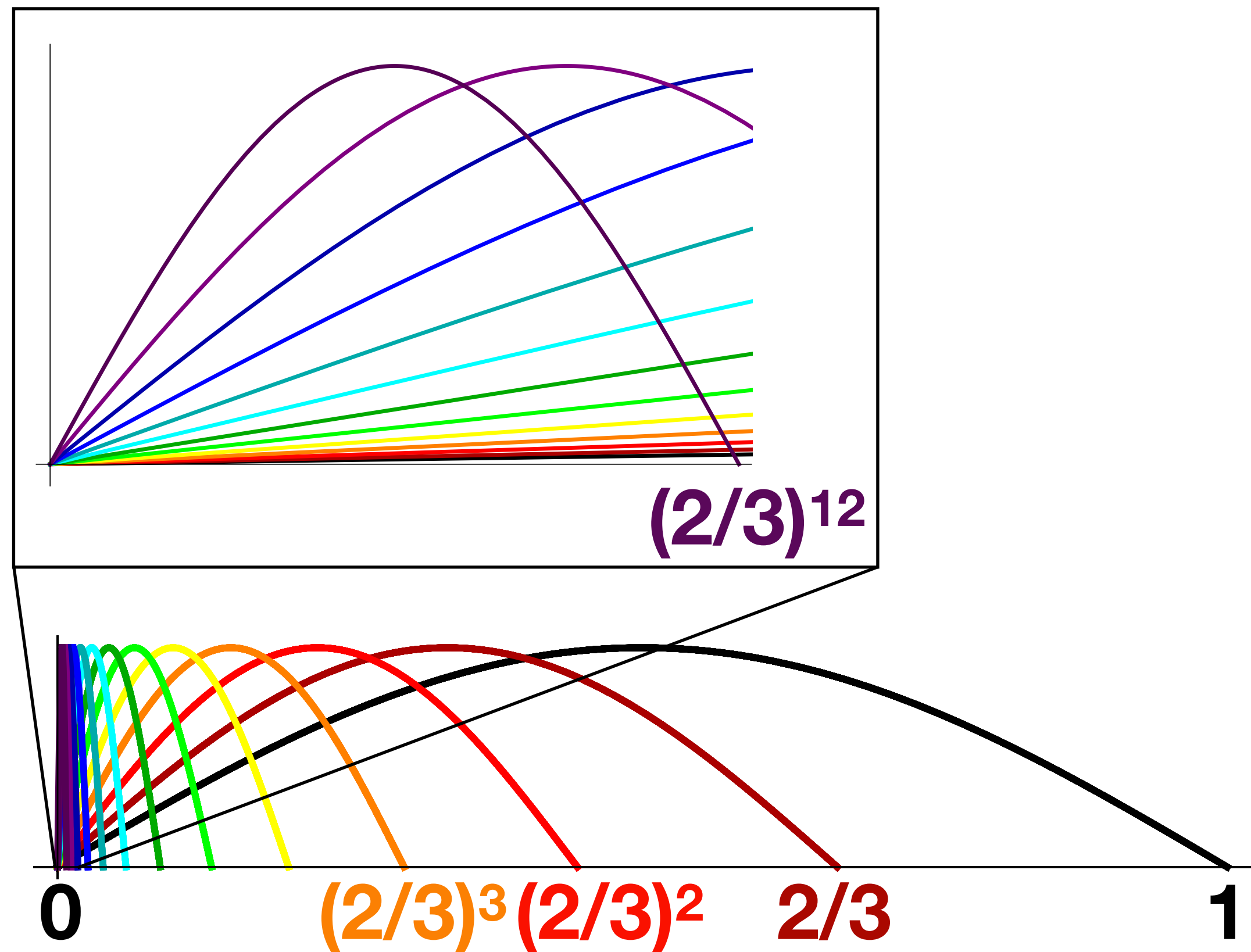


# Comparison to "cycle" of octaves

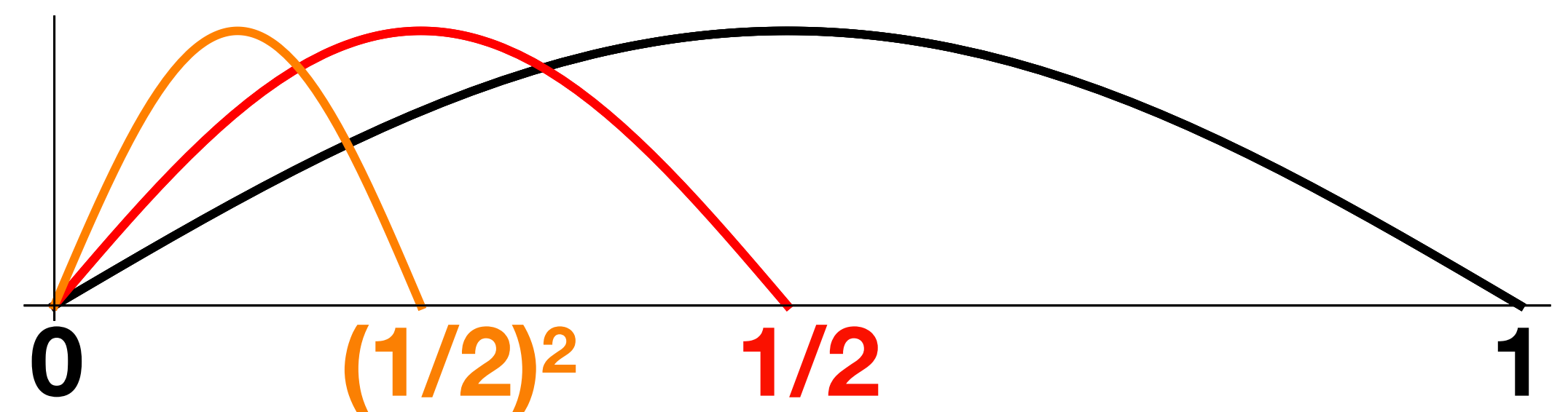




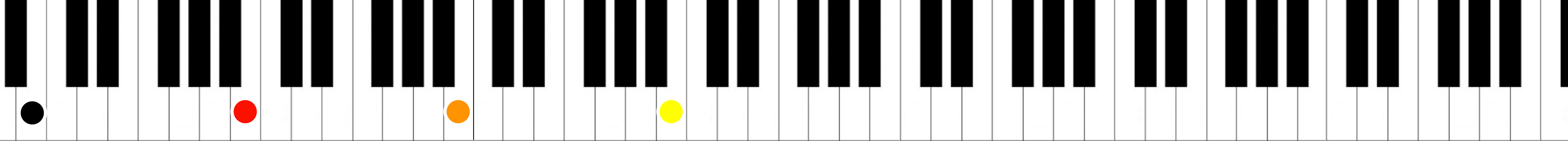
## Tuning following the cycle of fifths



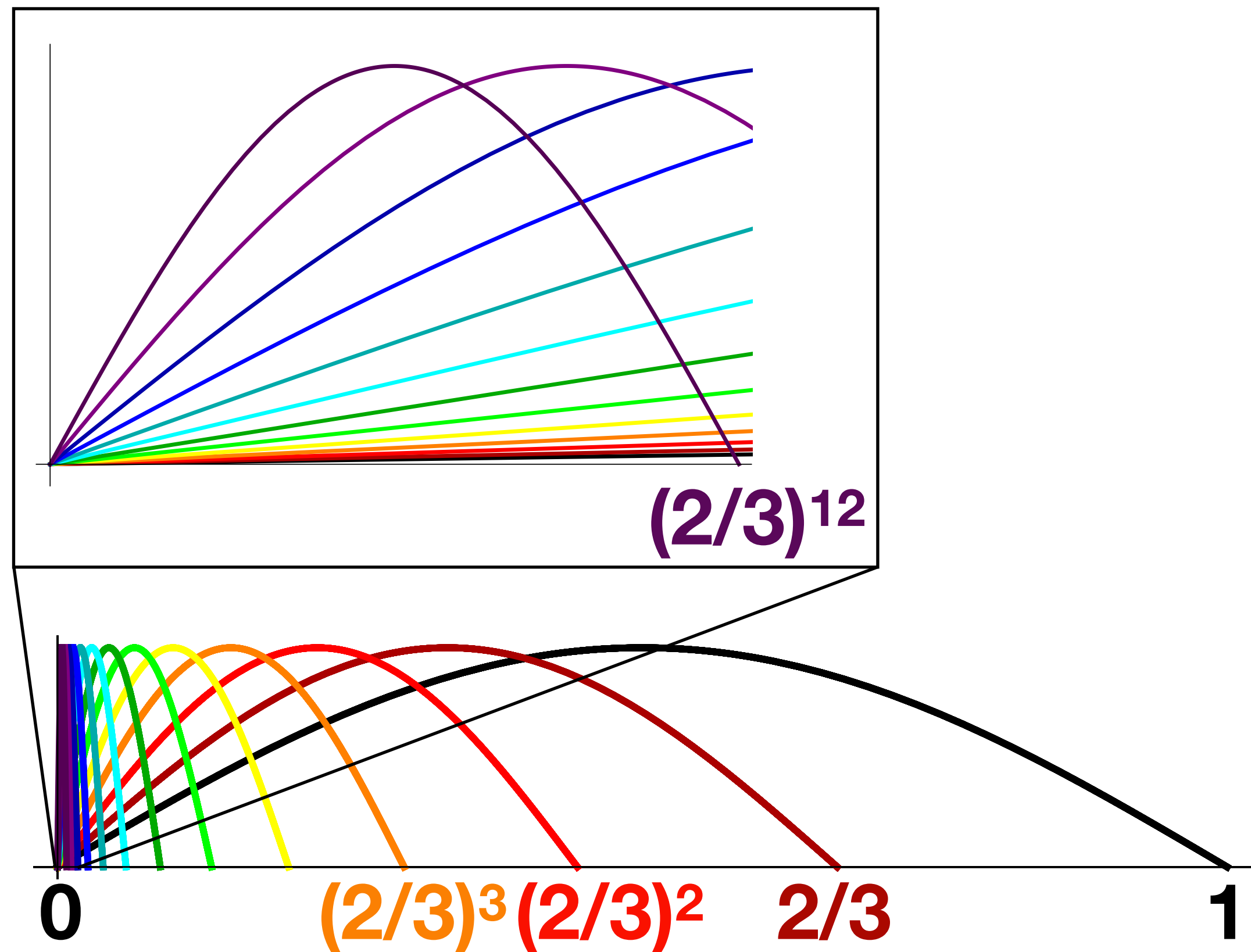
## Comparison to “cycle” of octaves



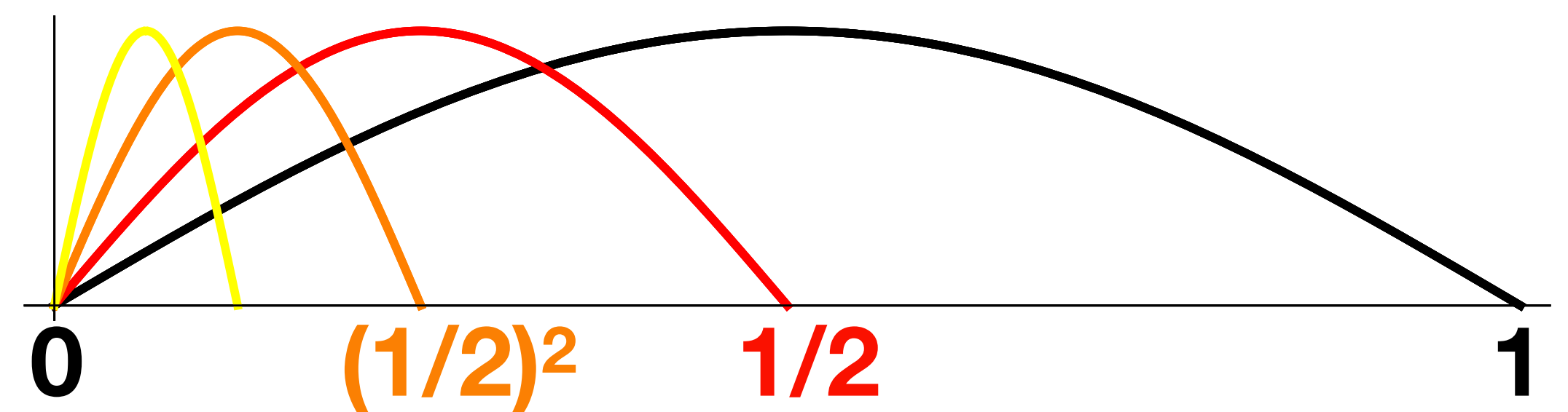


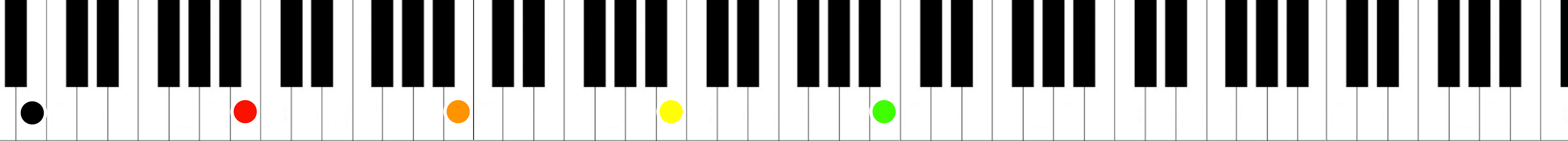


## Tuning following the cycle of fifths



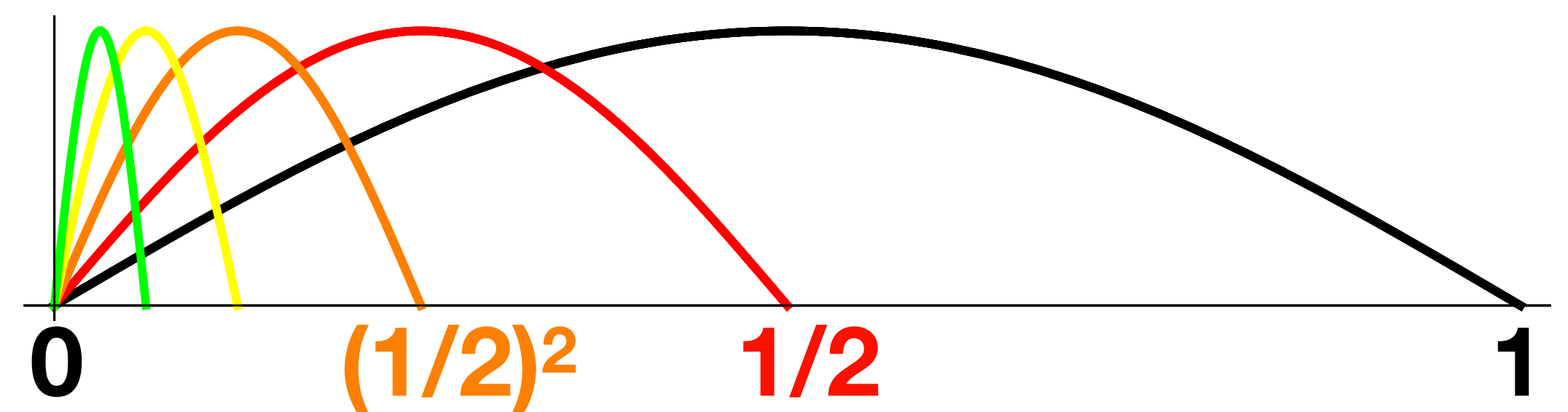
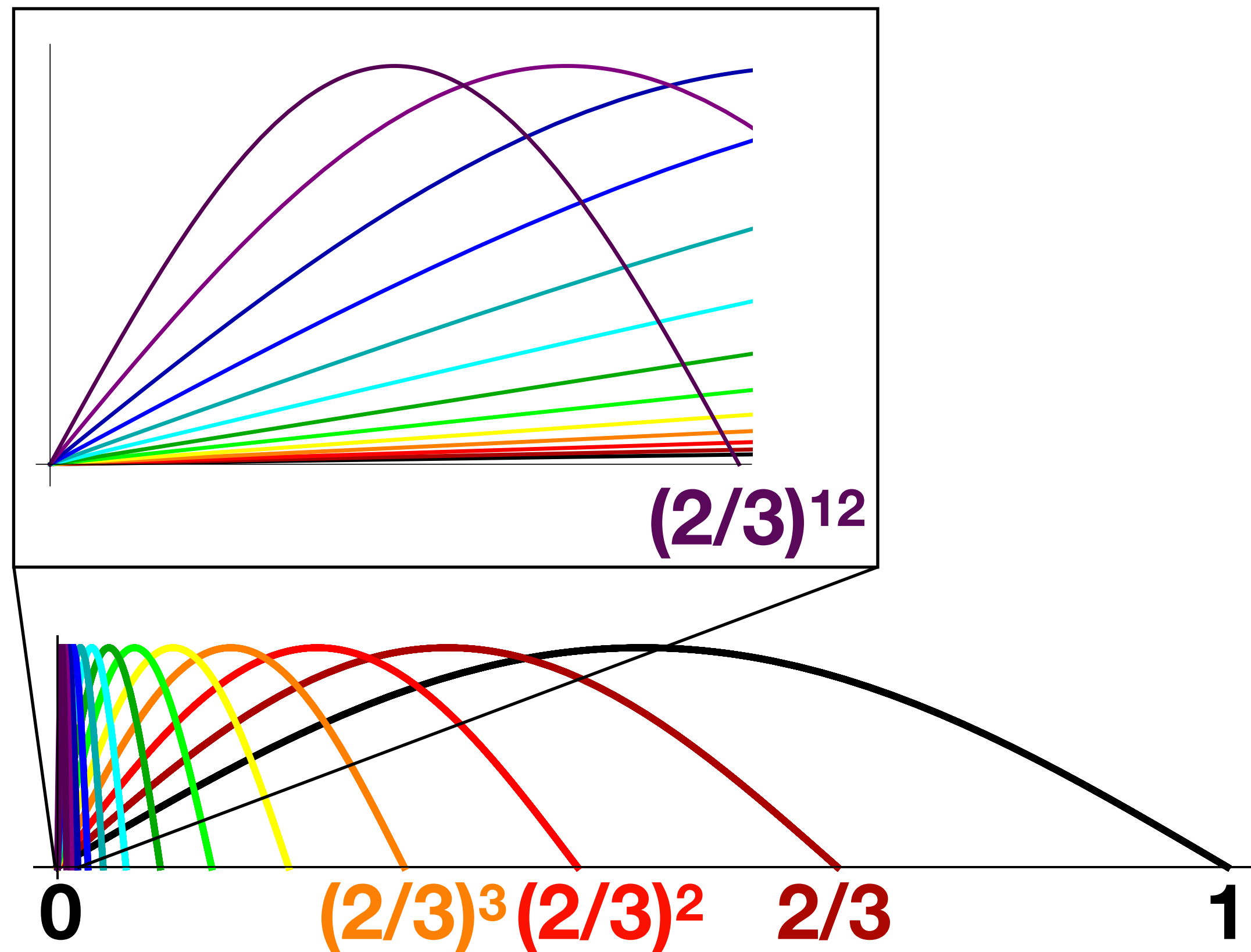
## Comparison to “cycle” of octaves

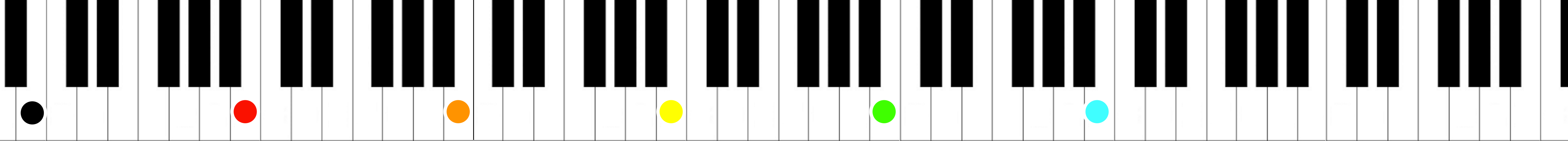




# Tuning following the cycle of fifths

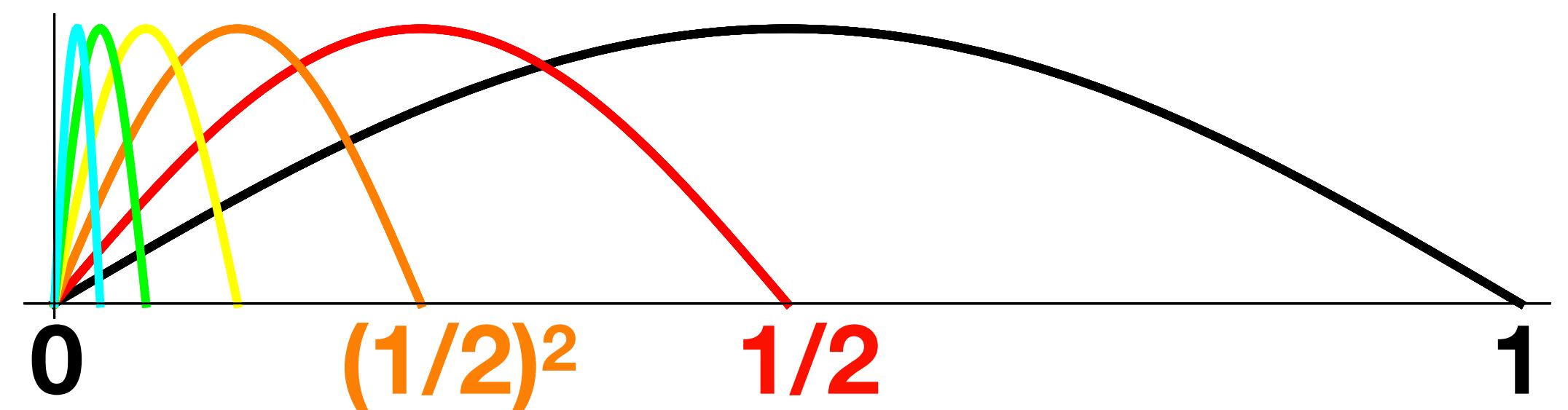
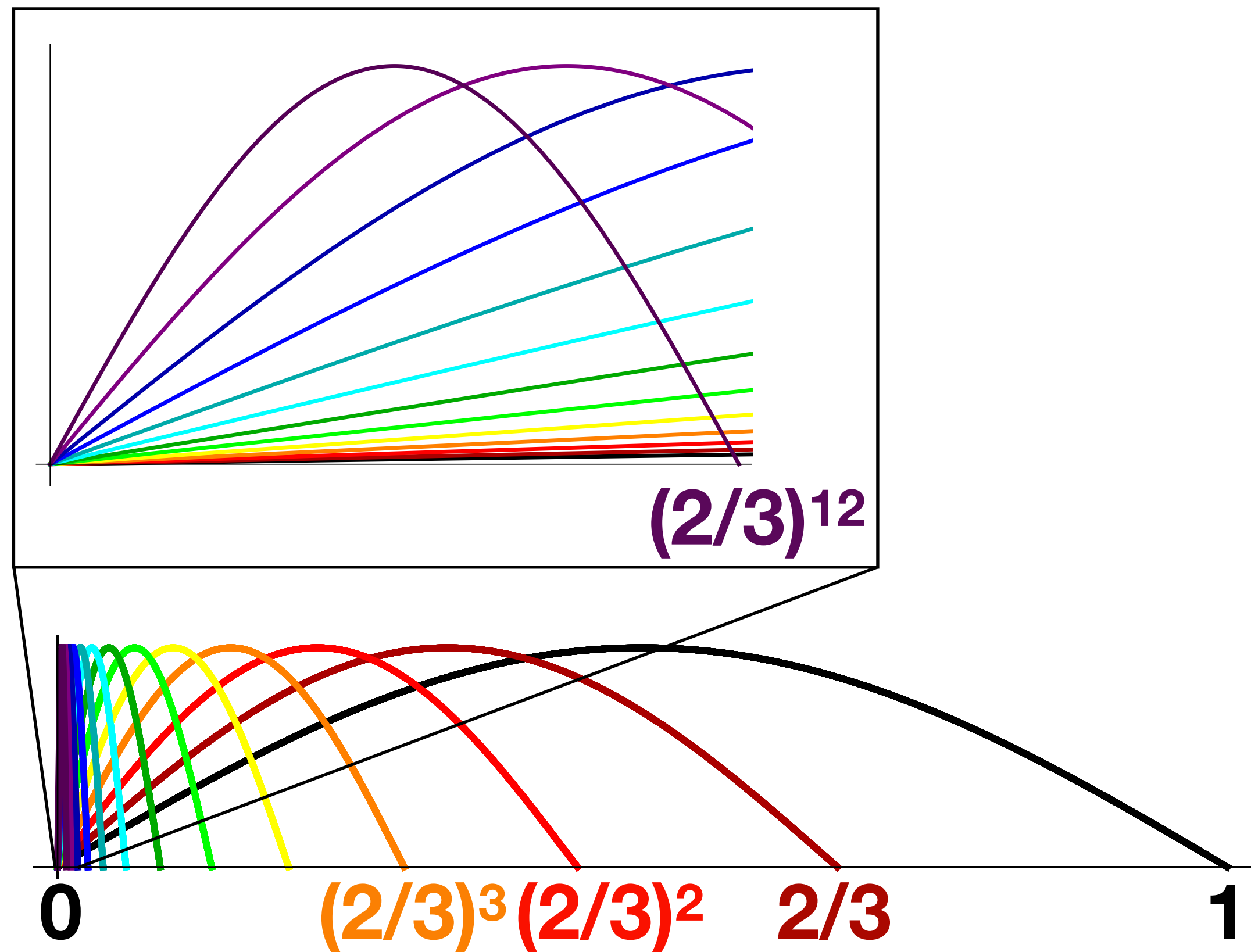
# Comparison to “cycle” of octaves

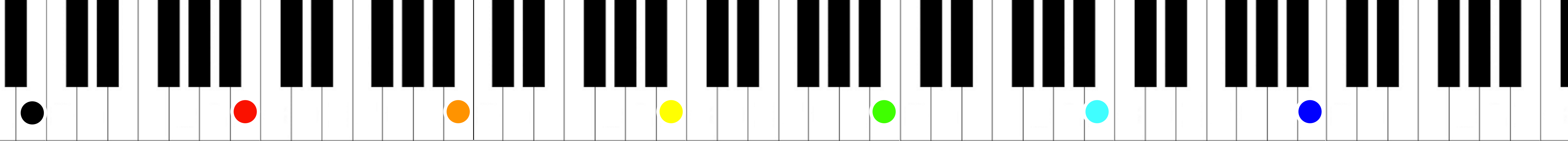




# Tuning following the cycle of fifths

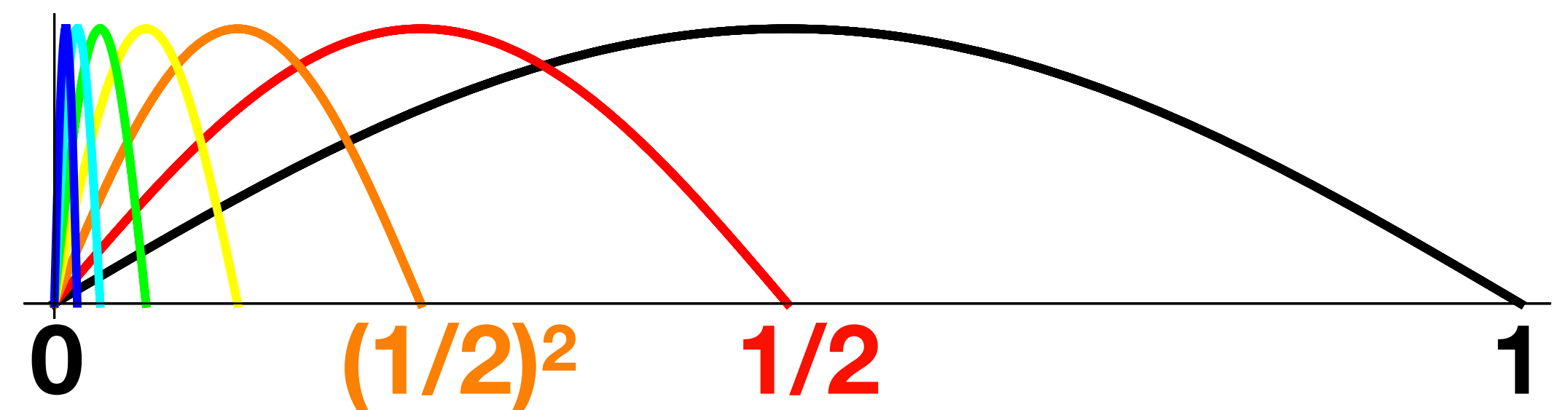
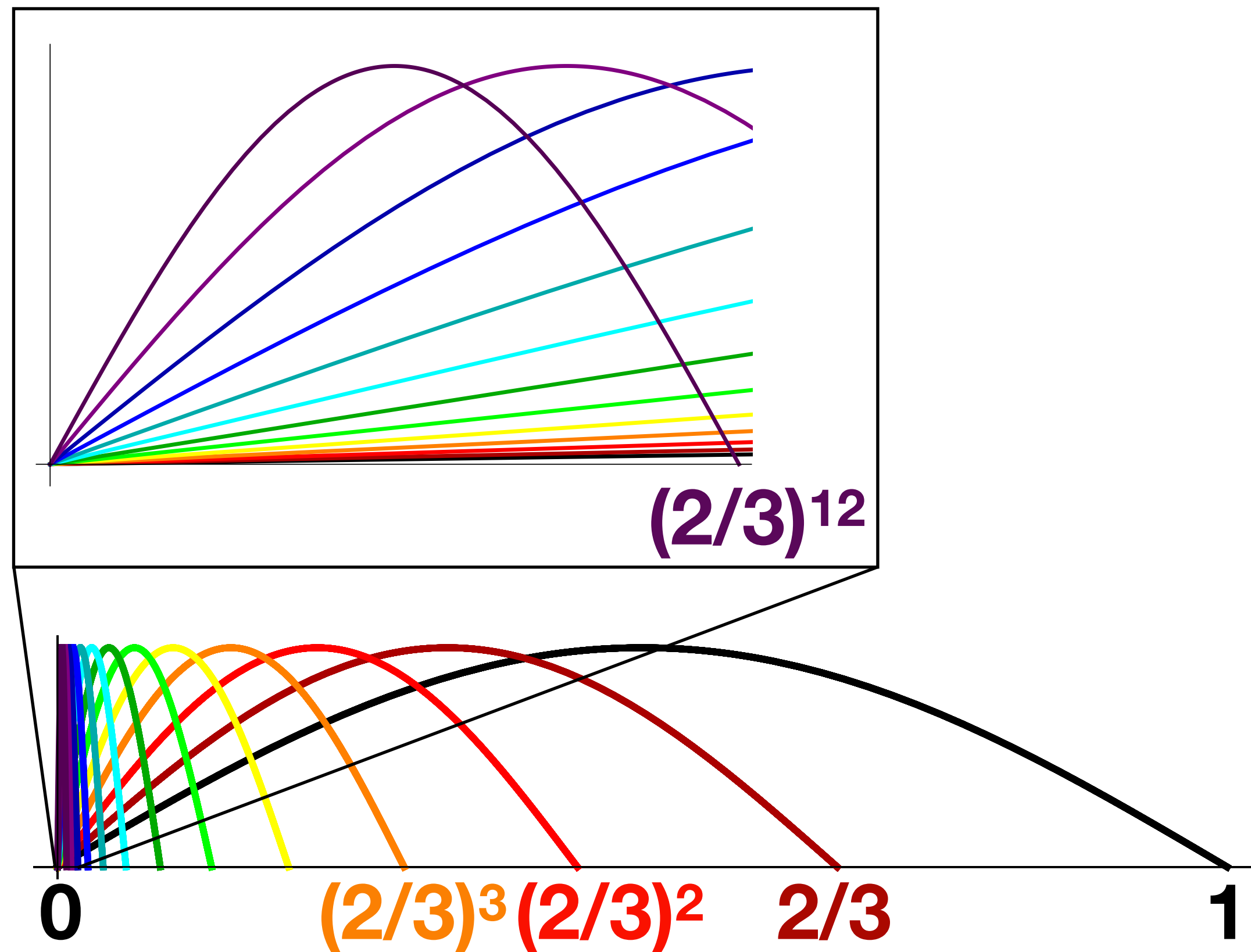
# Comparison to “cycle” of octaves

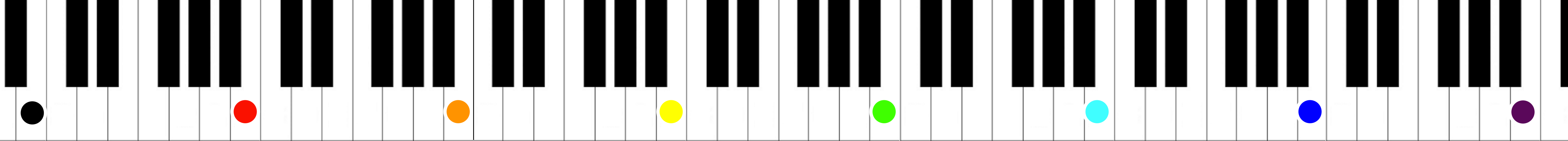




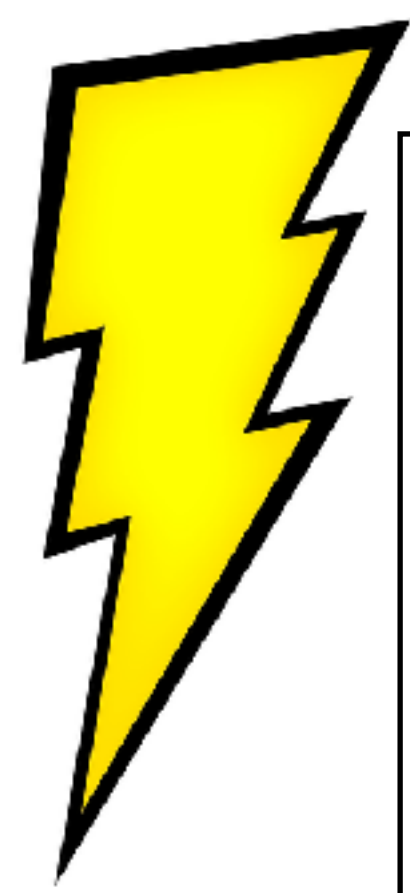
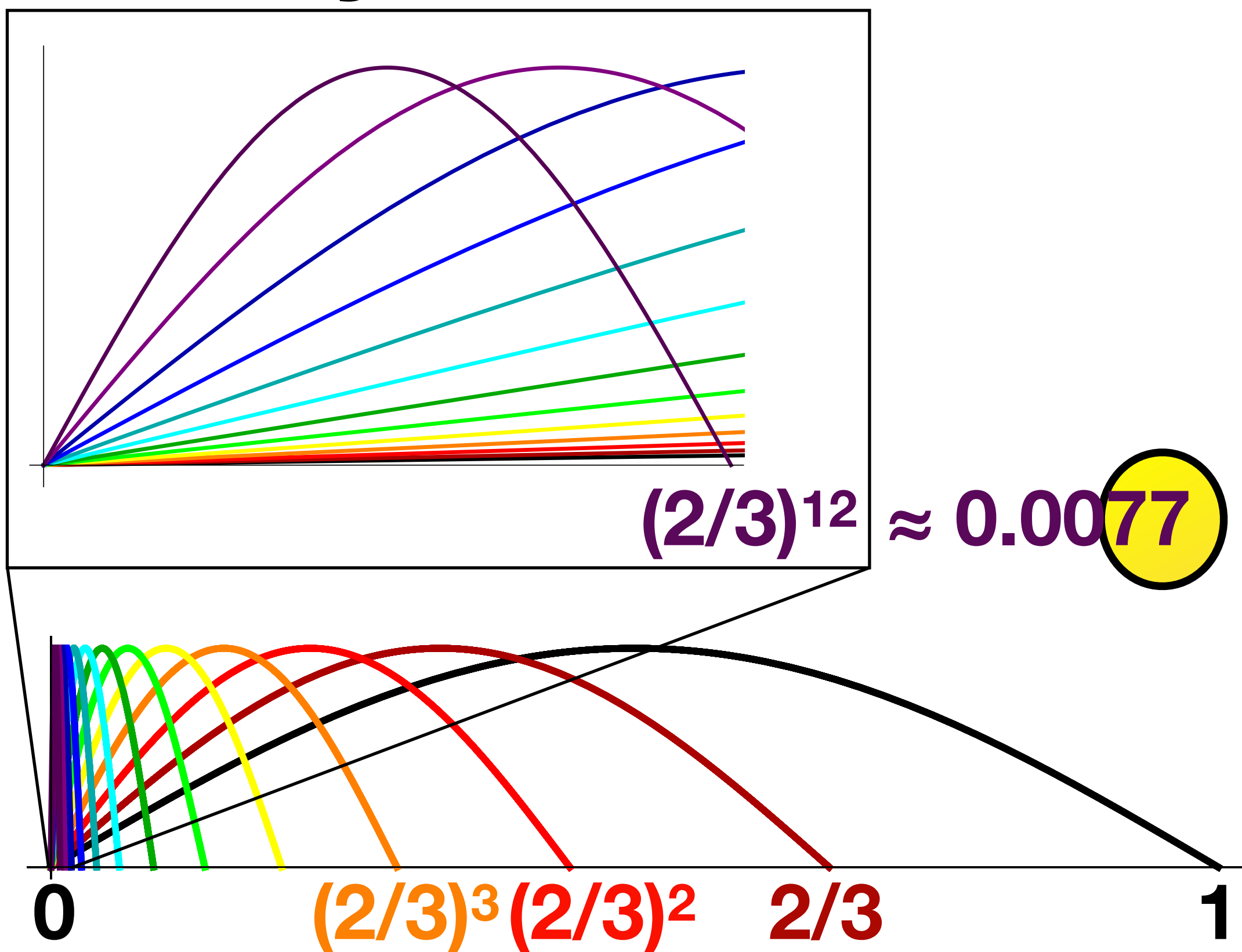
# Tuning following the cycle of fifths

# Comparison to “cycle” of octaves

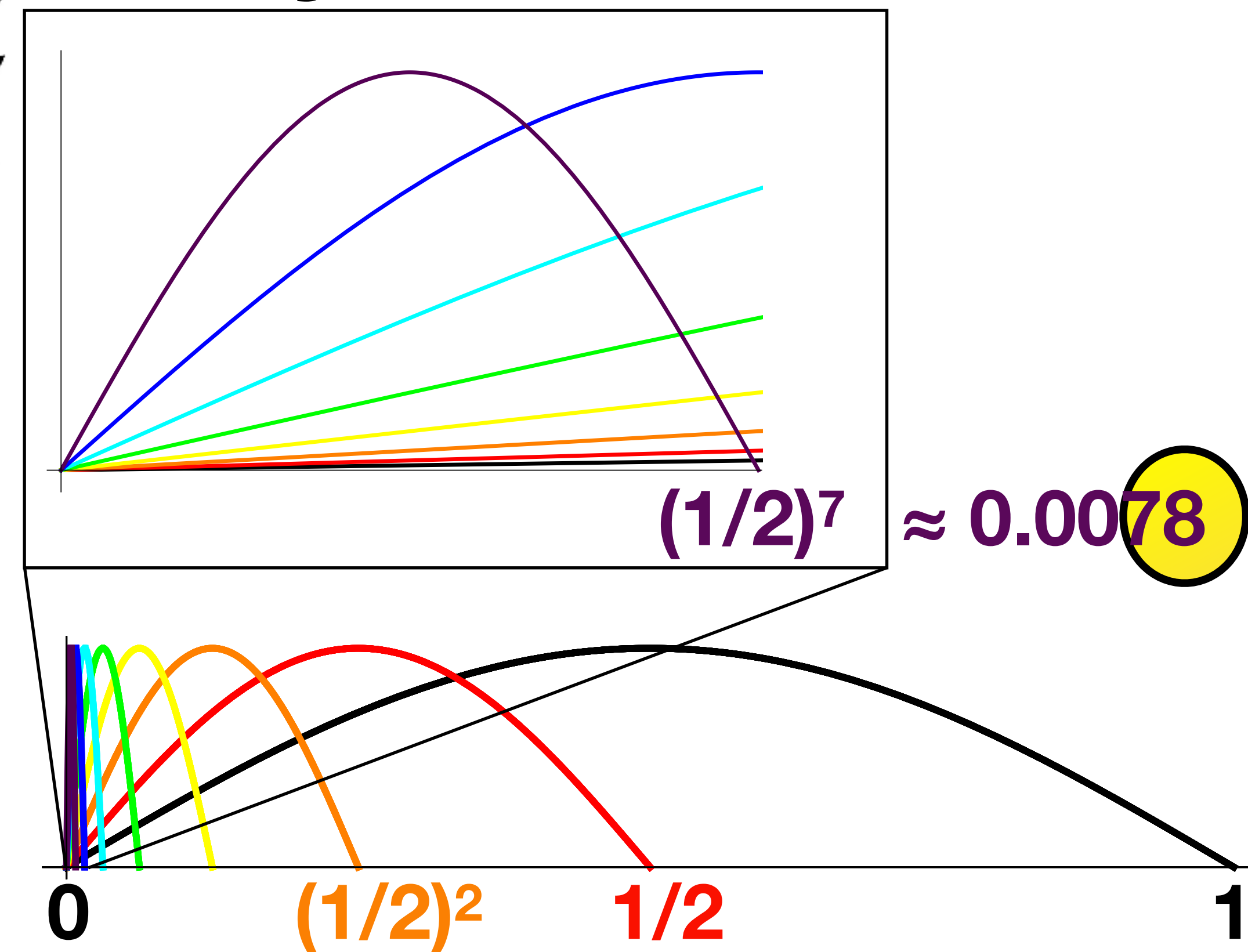




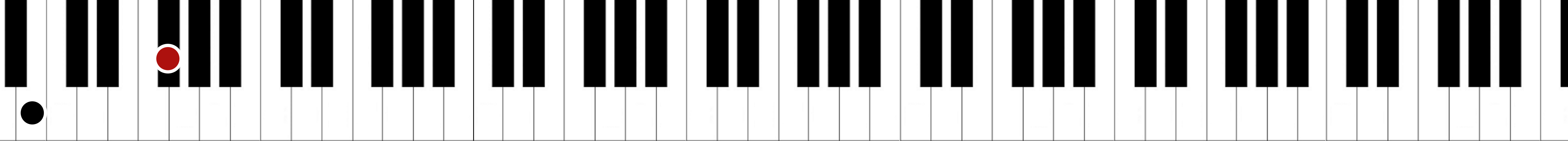
# Tuning following the cycle of fifths



# Comparison to "cycle" of octaves

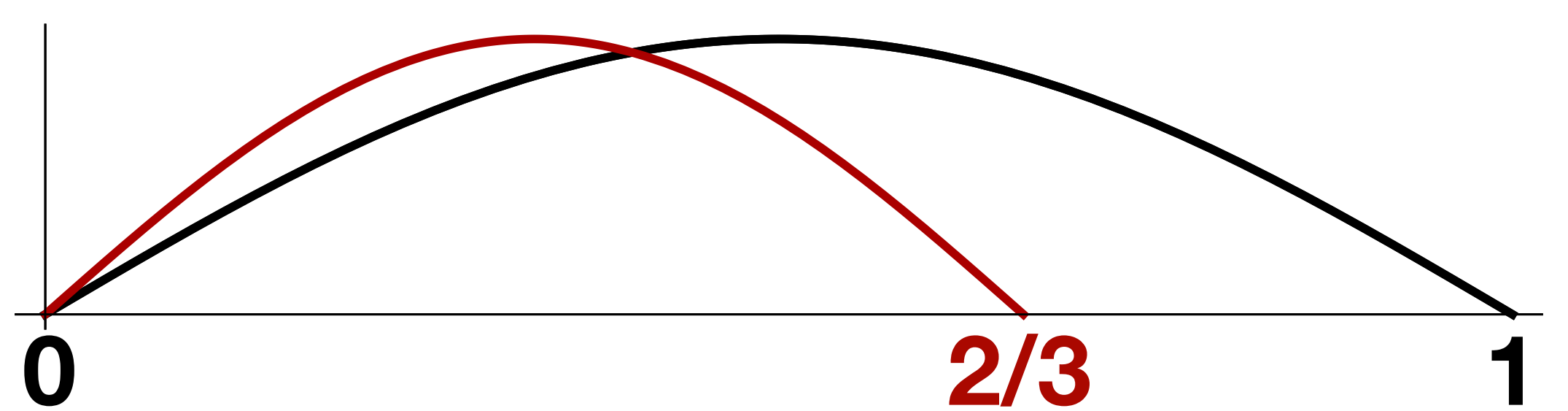
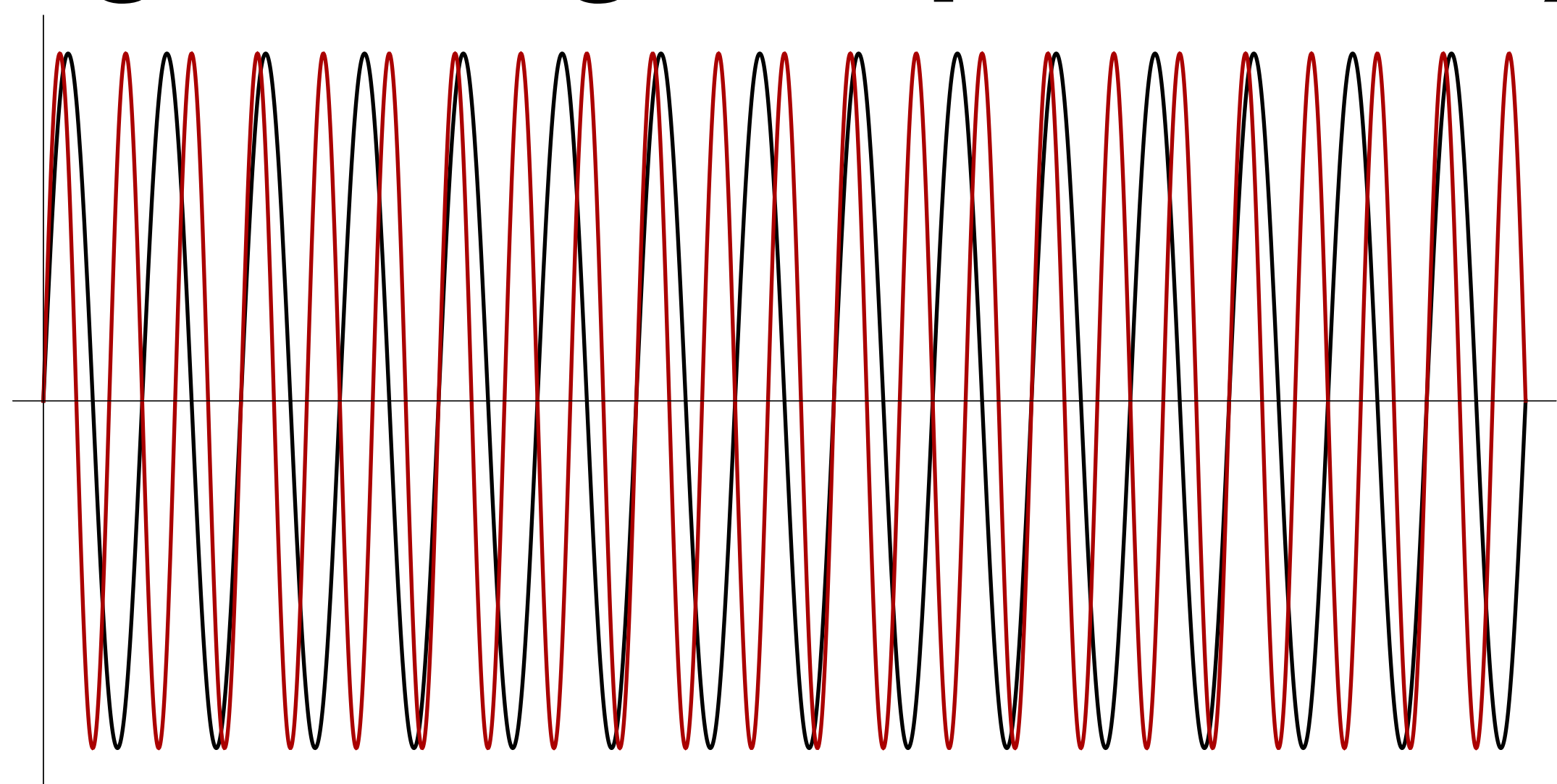




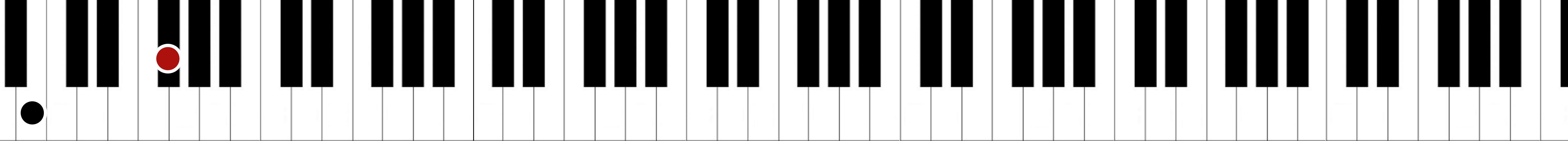


**commensurate intervals  
(e.g. Young temperament)**

**incommensurate intervals  
(equal temperament)**

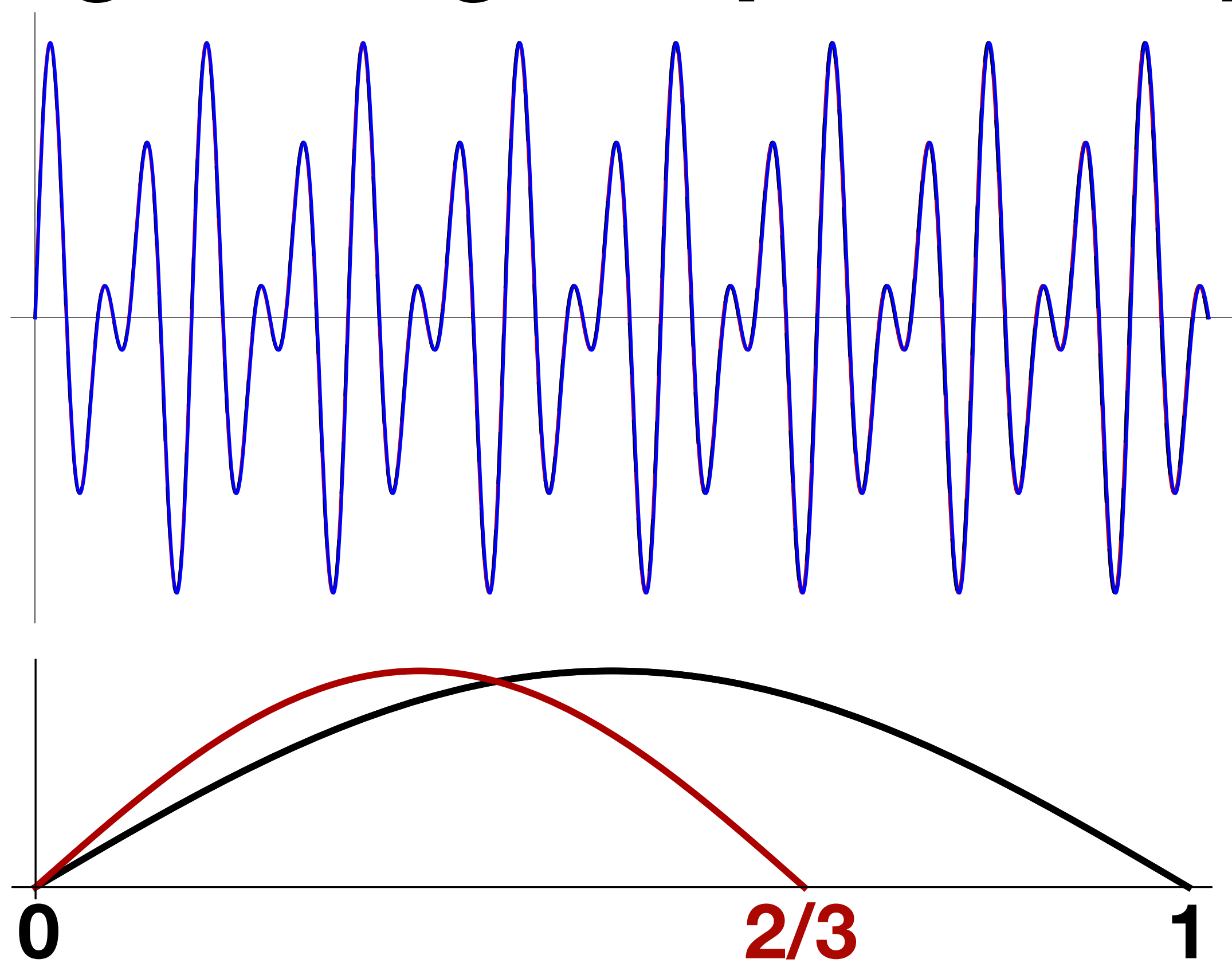


**(Pythagorean fifth)**

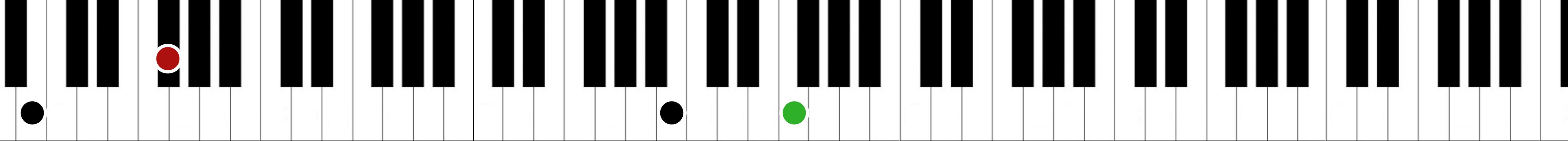


**commensurate intervals  
(e.g. Young temperament)**

**incommensurate intervals  
(equal temperament)**

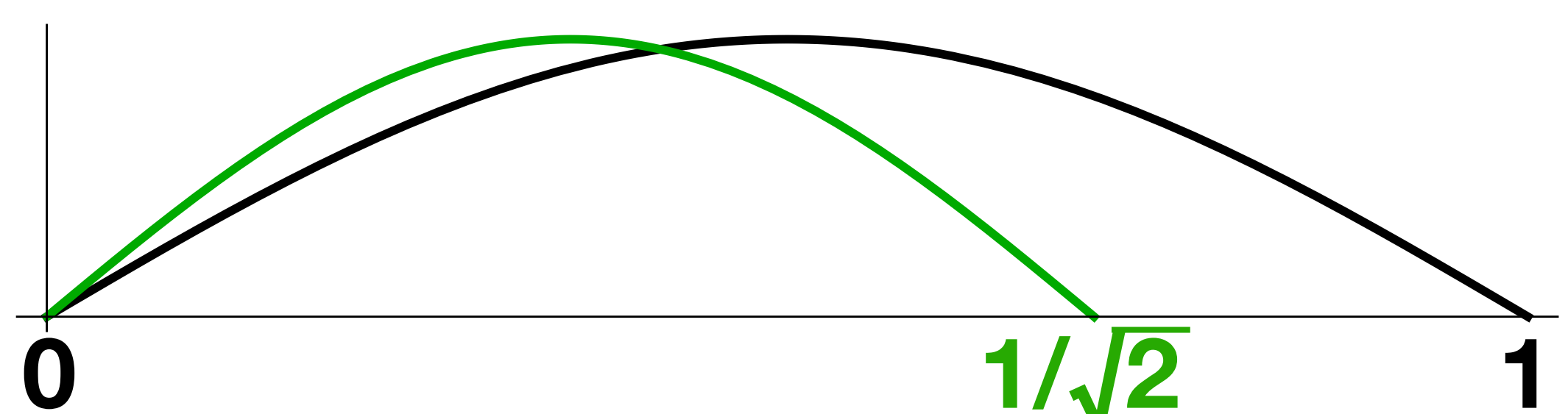
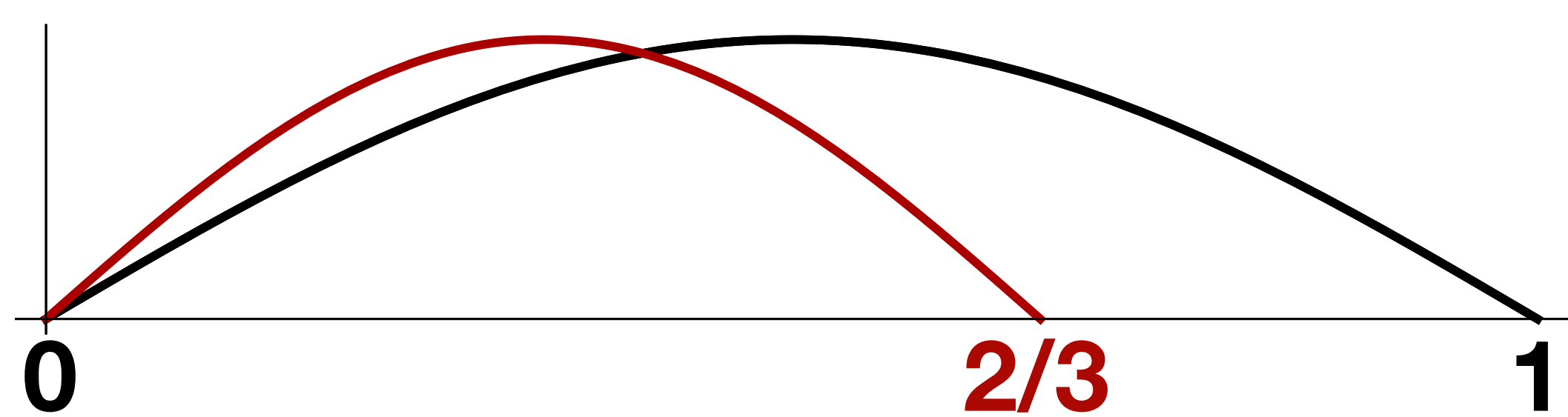
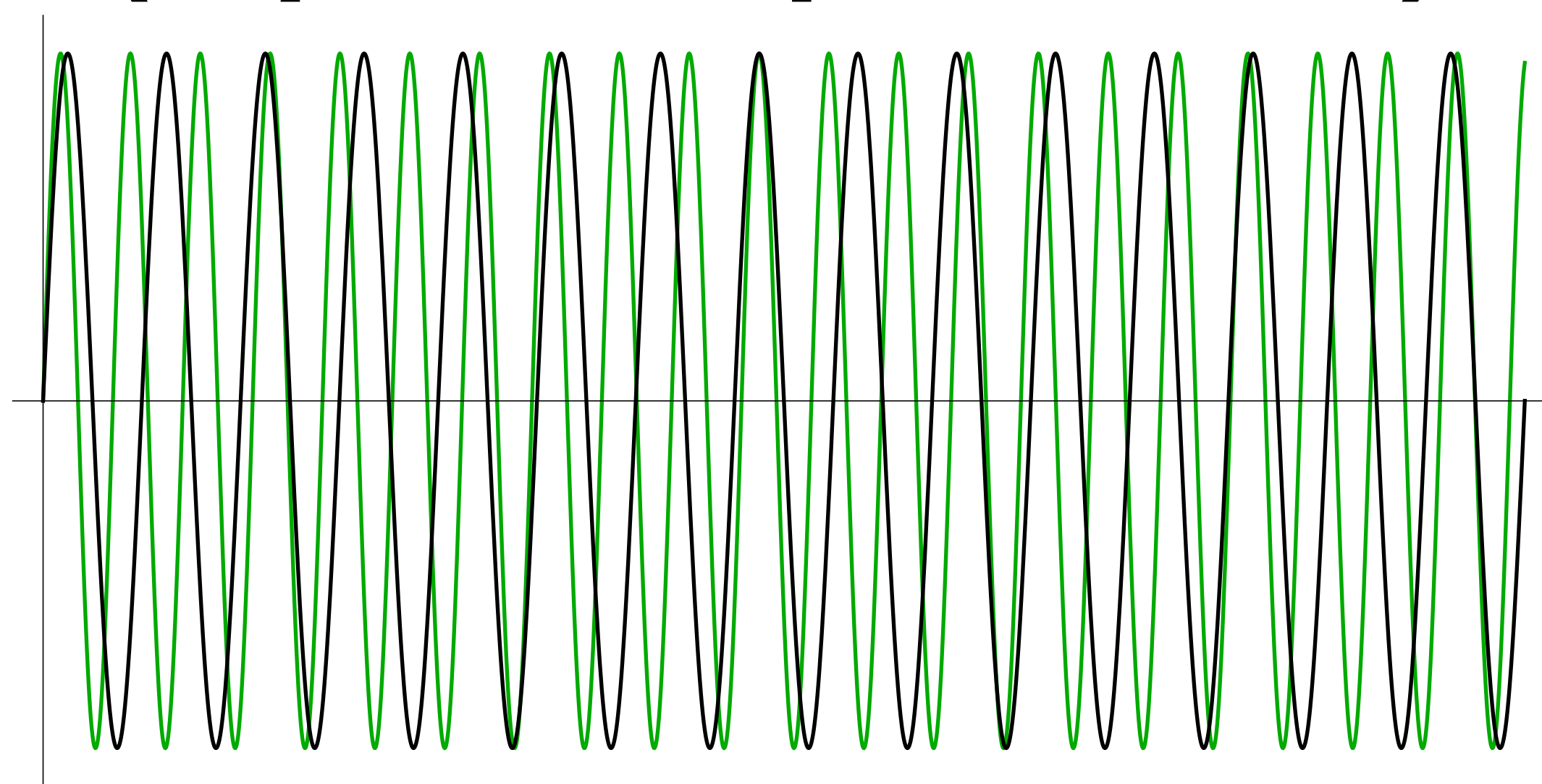
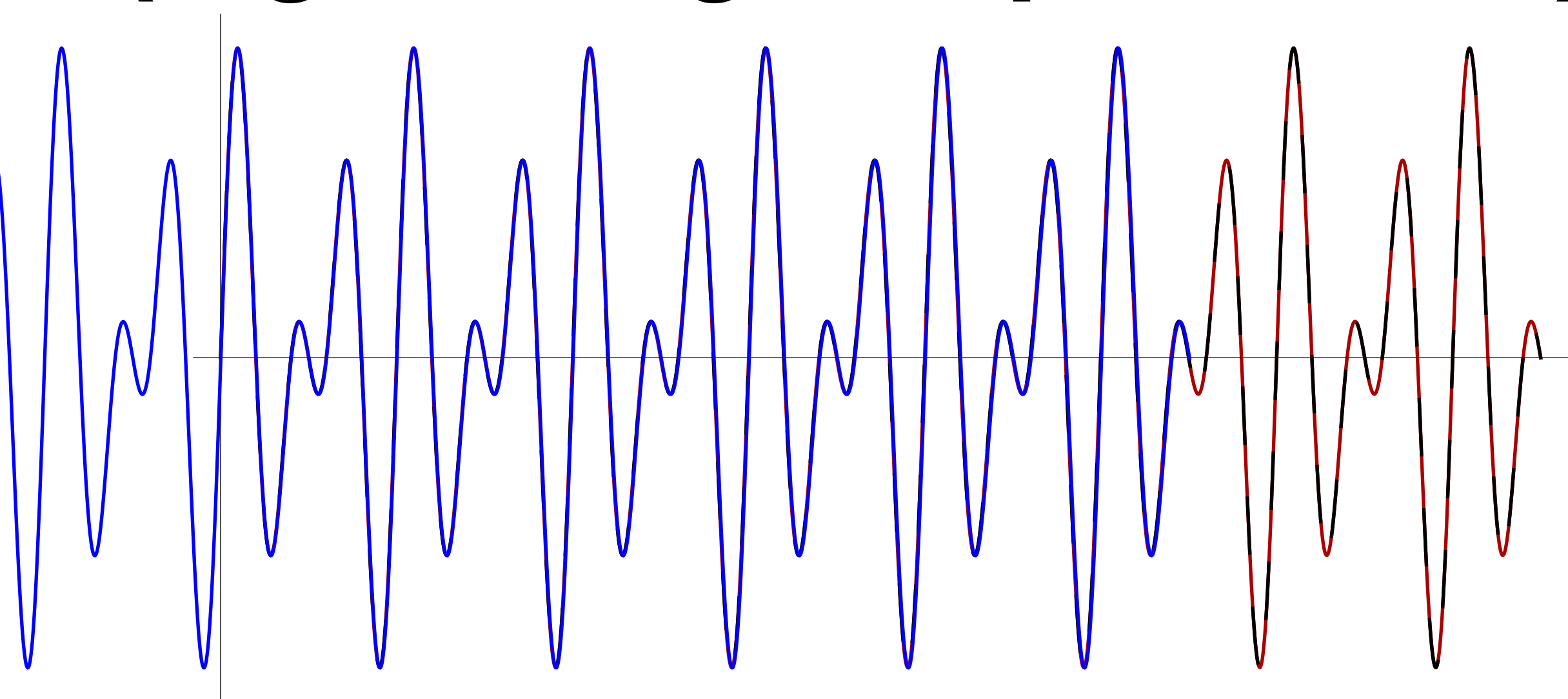


**(Pythagorean fifth)**



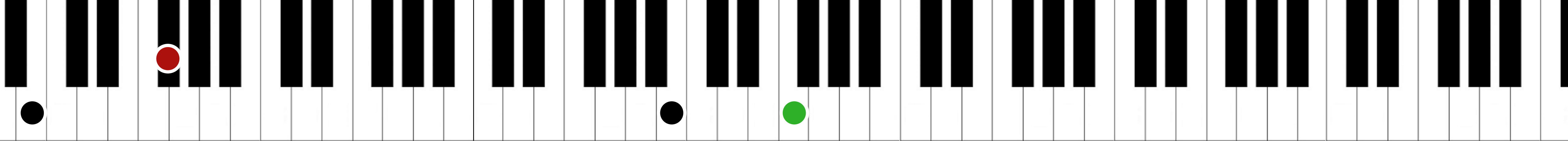
**commensurate intervals  
(e.g. Young temperament)**

**incommensurate intervals  
(equal temperament)**



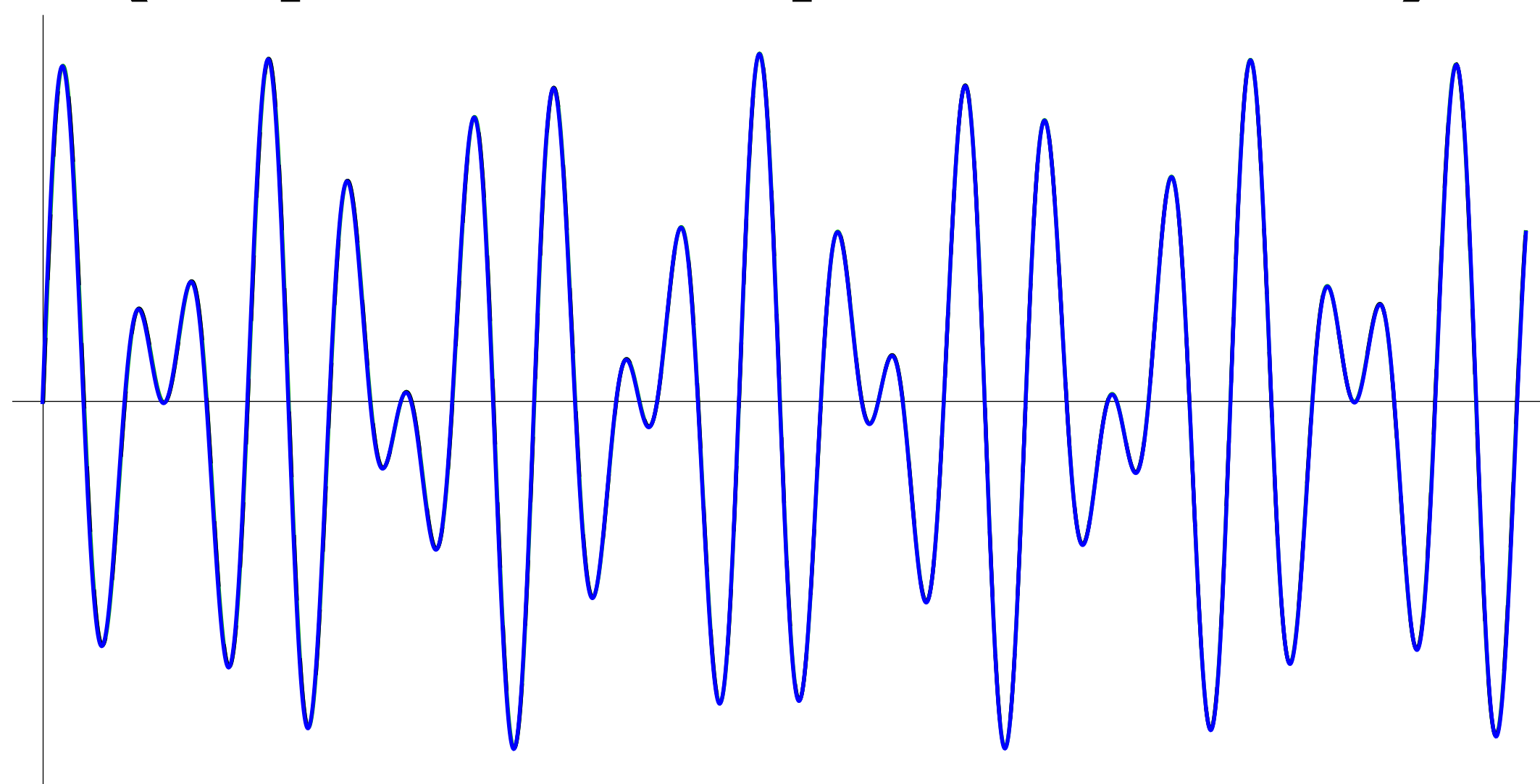
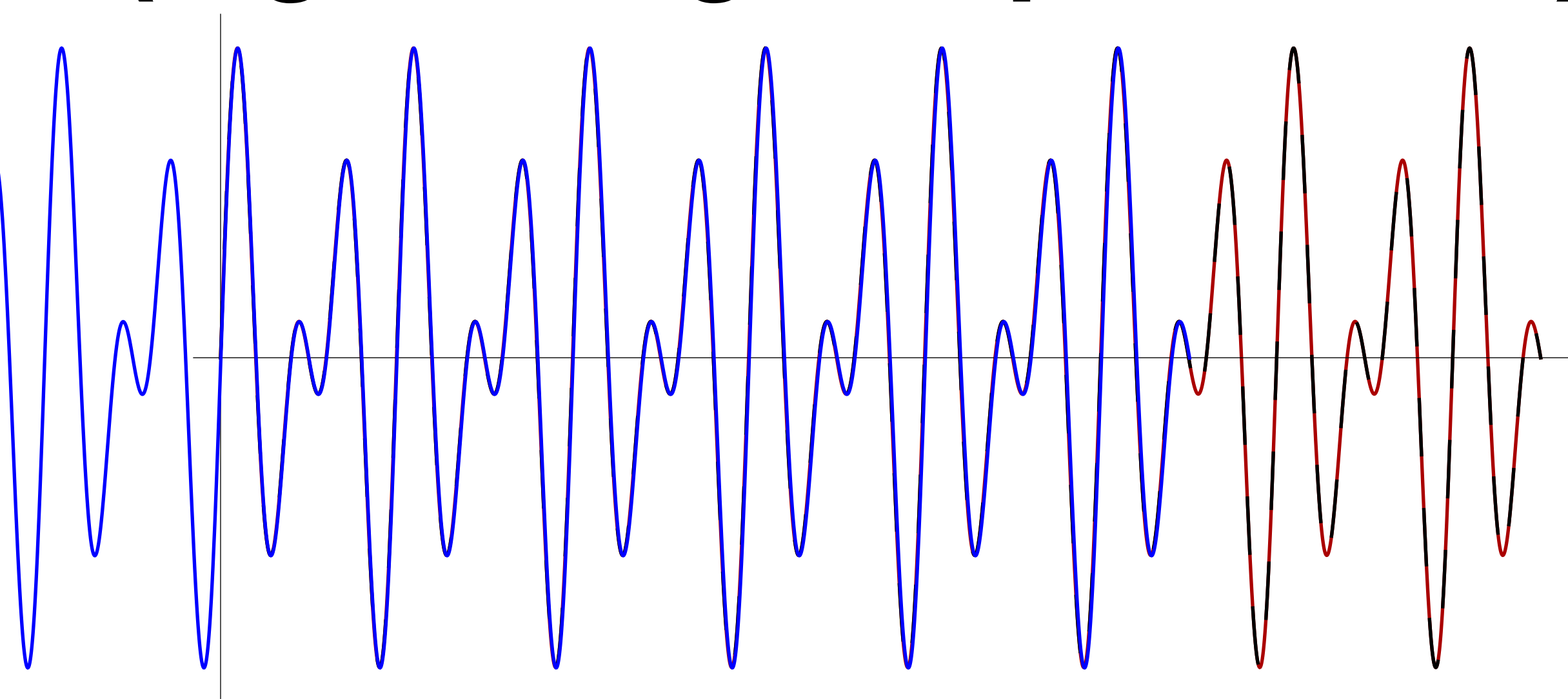
**(Pythagorean fifth)**

**(Equal temperament tritone)**



**commensurate intervals  
(e.g. Young temperament)**

**incommensurate intervals  
(equal temperament)**



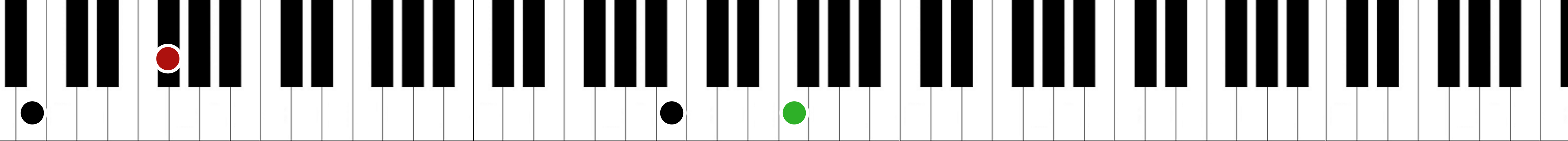
0  $\frac{2}{3}$  1

0  $\frac{1}{\sqrt{2}}$  1

**(Pythagorean fifth)**

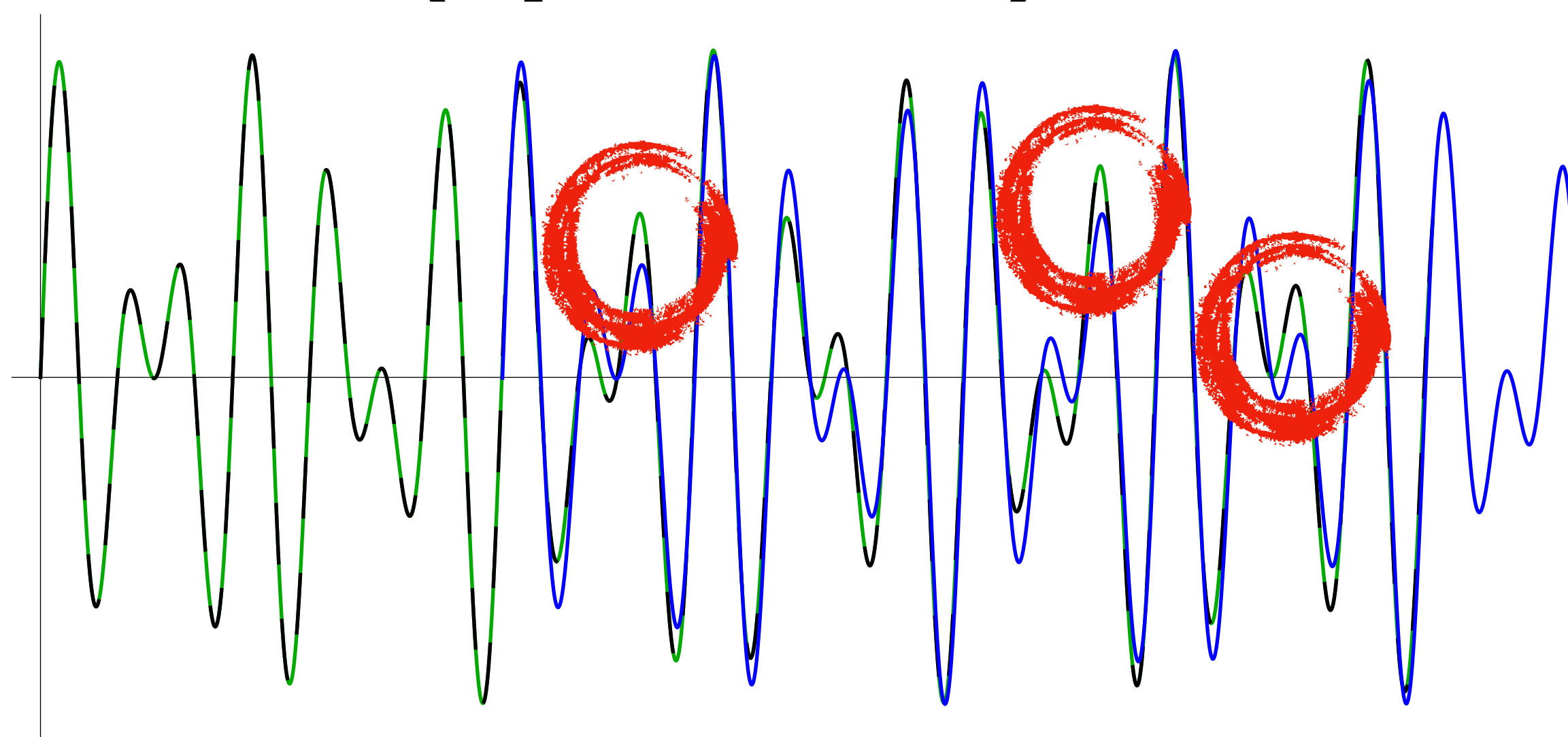
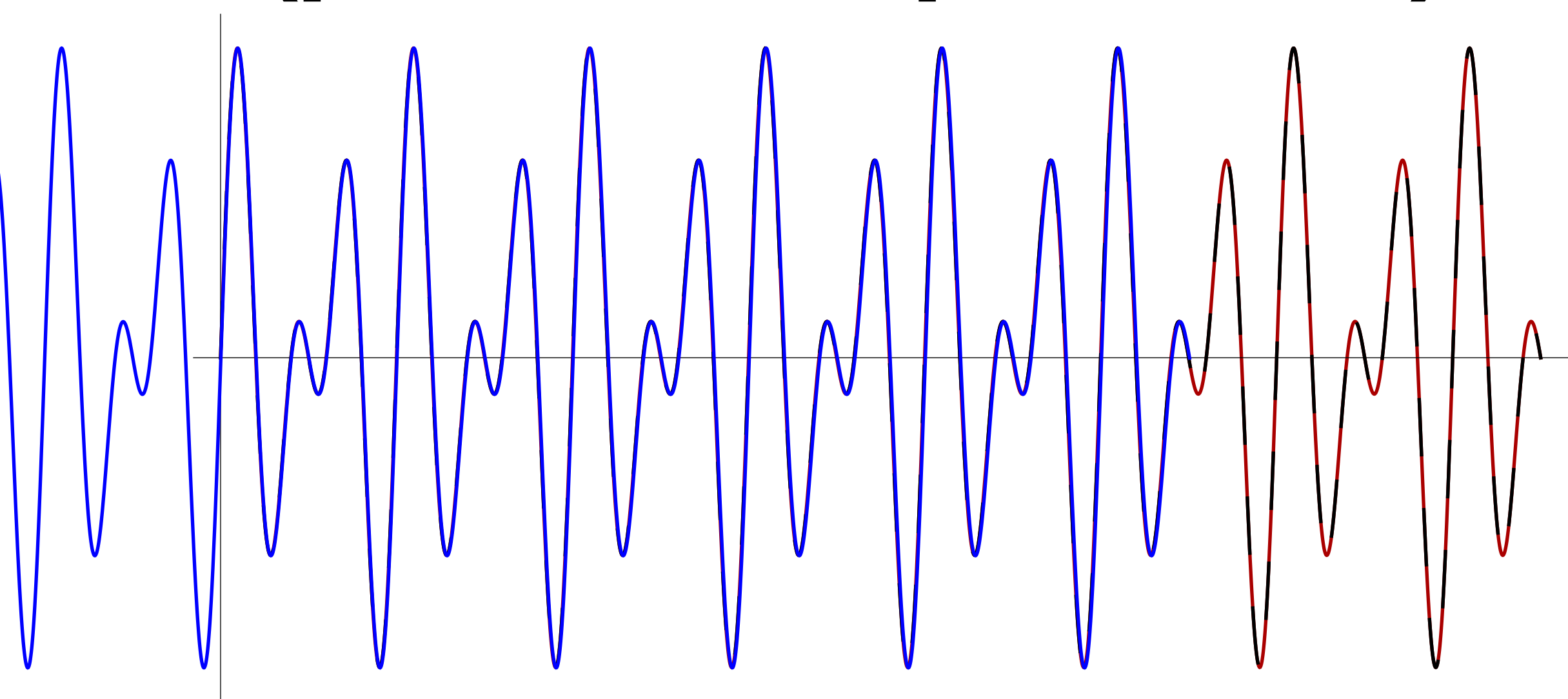
**(Equal temperament tritone)**





**commensurate intervals  
(periodic repetitions)**

**incommensurate intervals  
(aperiodic)**



0  $\frac{2}{3}$  1

0  $\frac{1}{\sqrt{2}}$  1

**(Pythagorean fifth)**

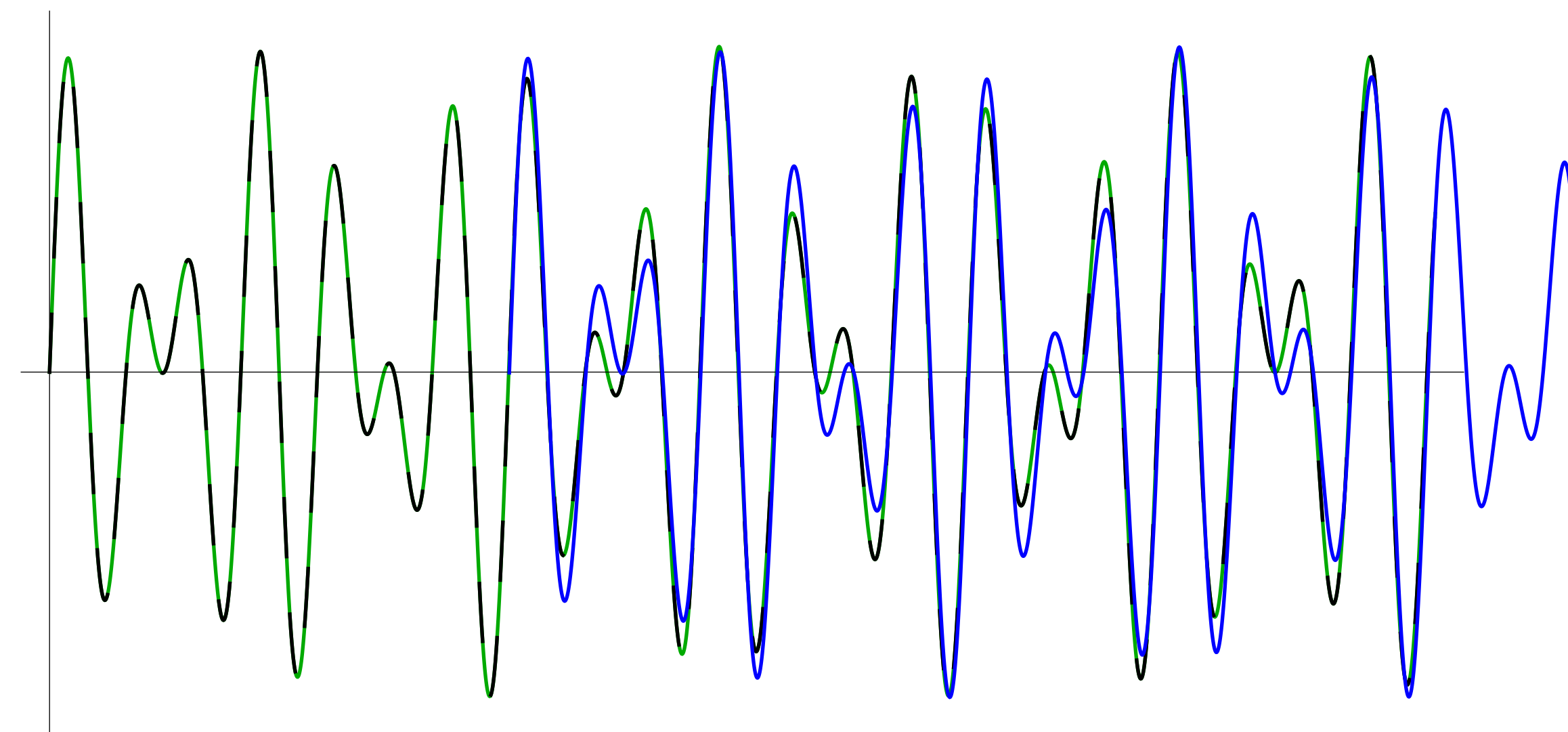
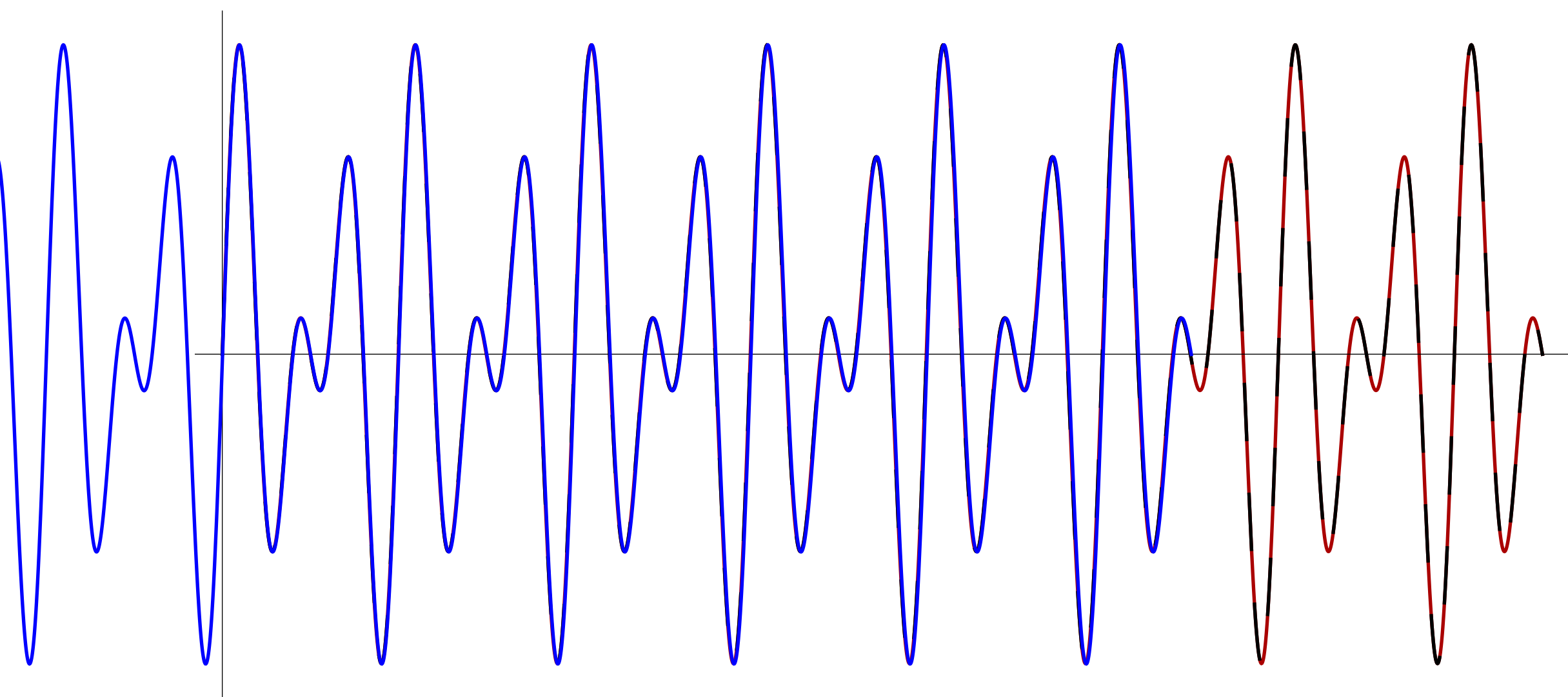
**(Equal temperament tritone)**



# Quantum particles in...

...commensurate intervals  
(periodic repetitions)

...incommensurate intervals  
(aperiodic)





# Quantum particles in...

...commensurate intervals  
(periodic repetitions)

...incommensurate intervals  
(aperiodic)



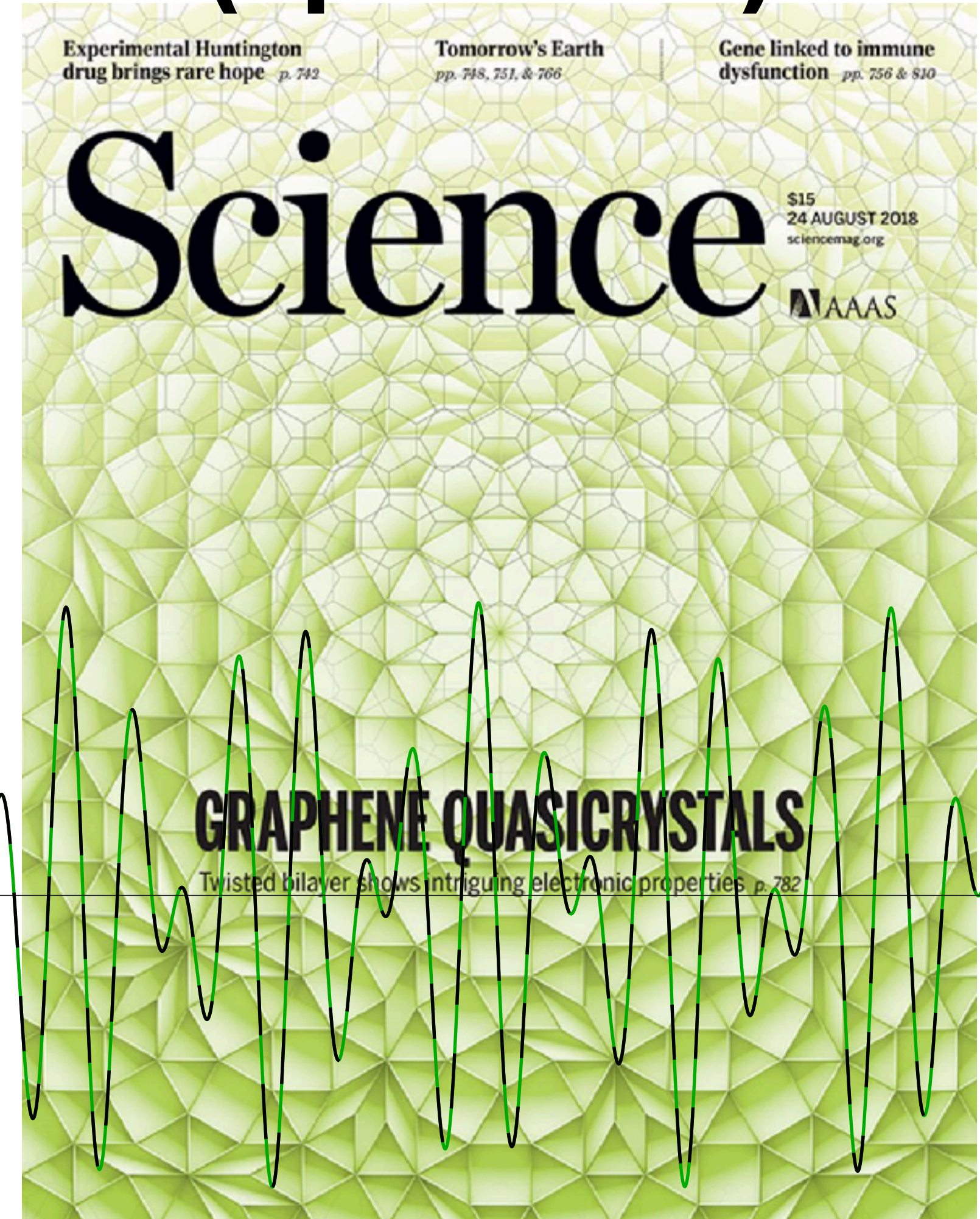


# Quantum particles in...

...crystals  
(periodic repetitions)



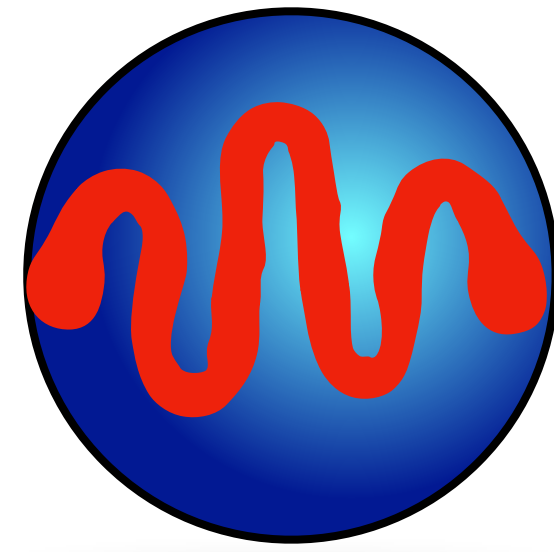
...quasicrystals  
(aperiodic)



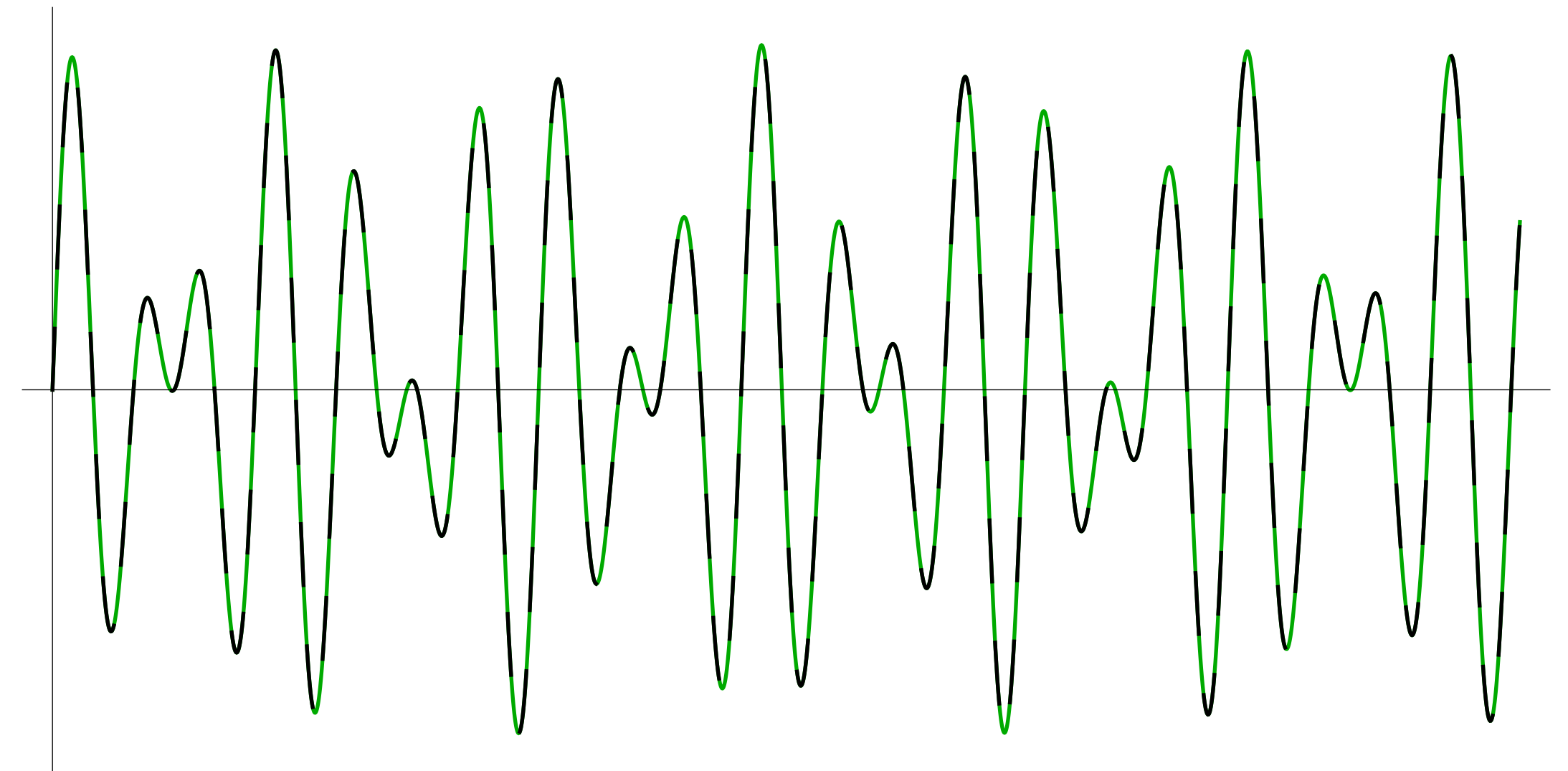
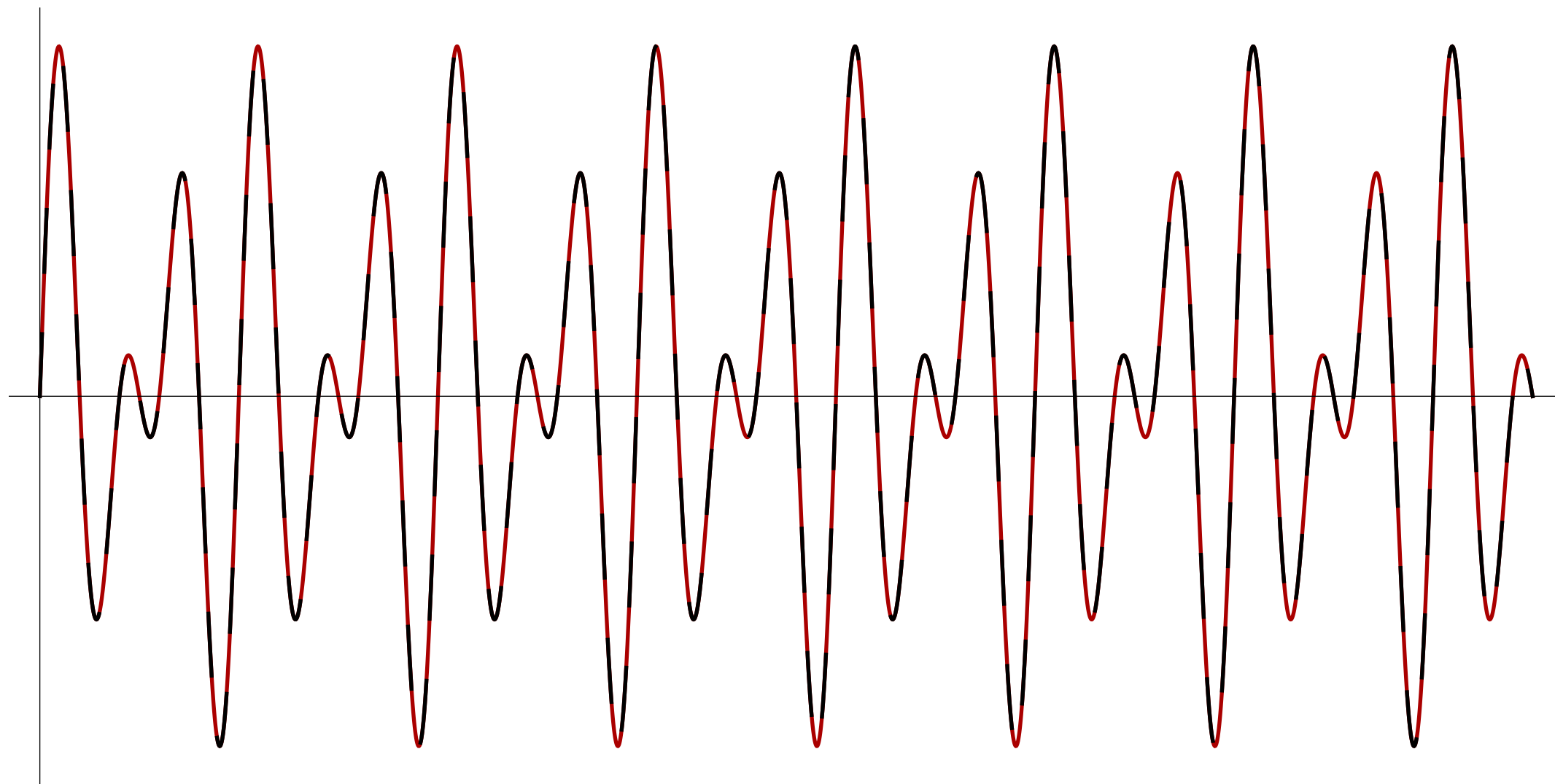


# Quantum particles in...

**...crystals  
(periodic repetitions)**

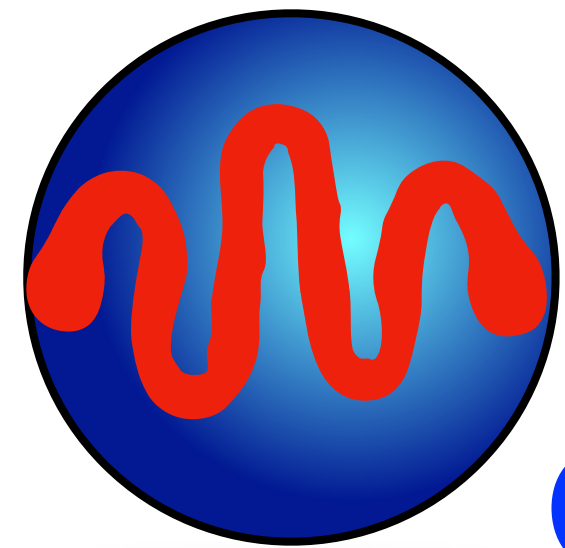


**...quasicrystals  
(aperiodic)**



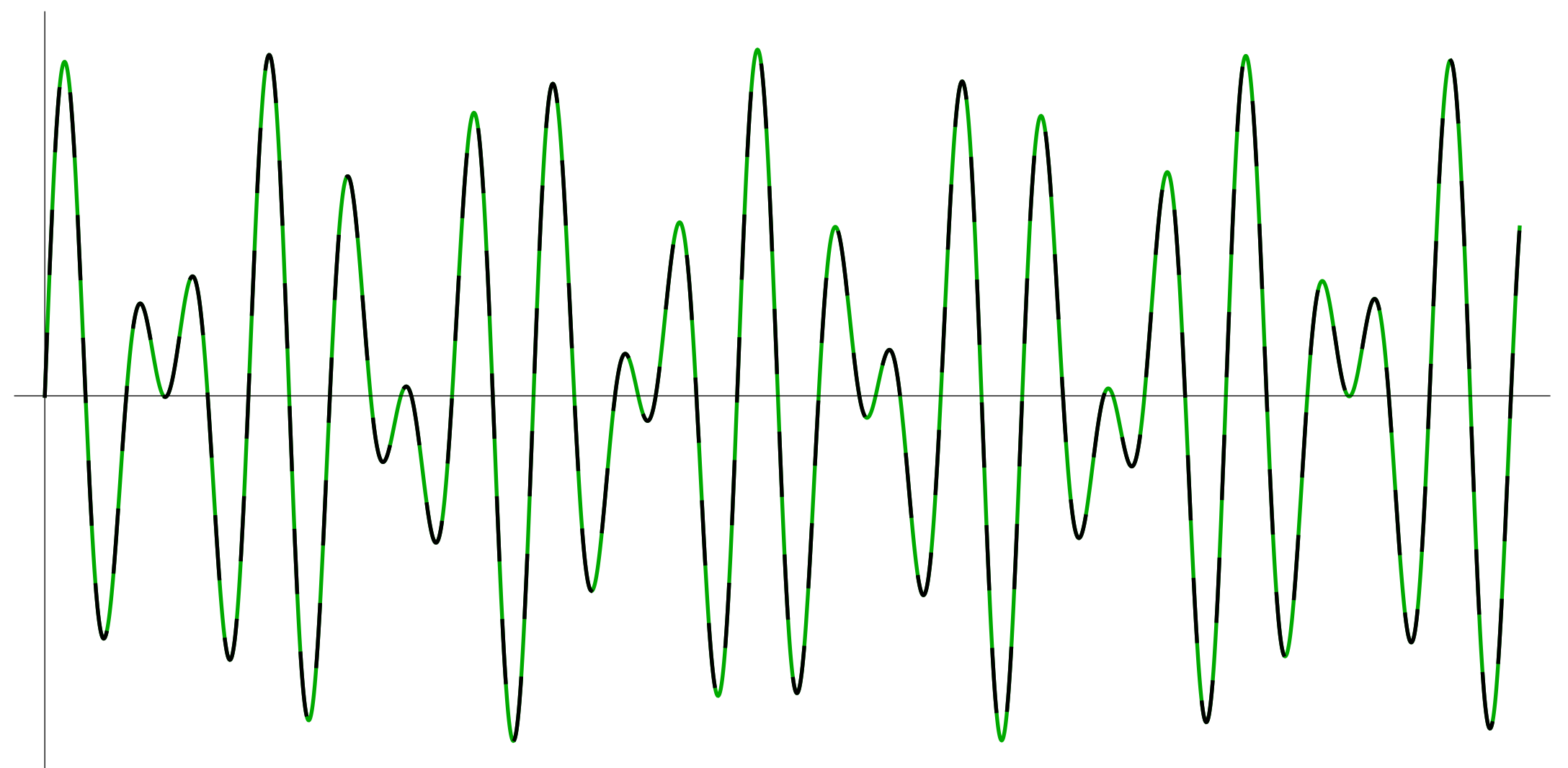
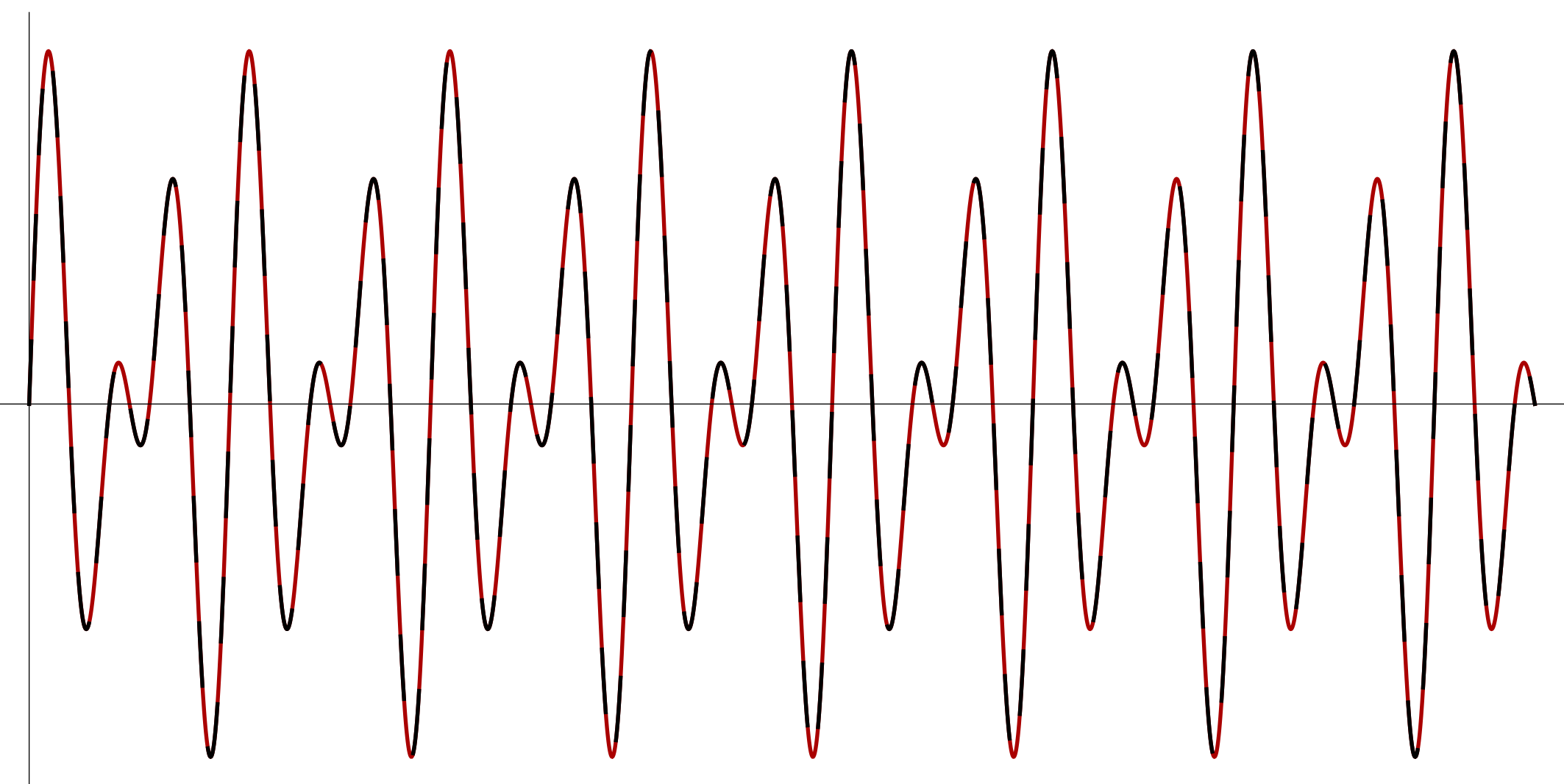
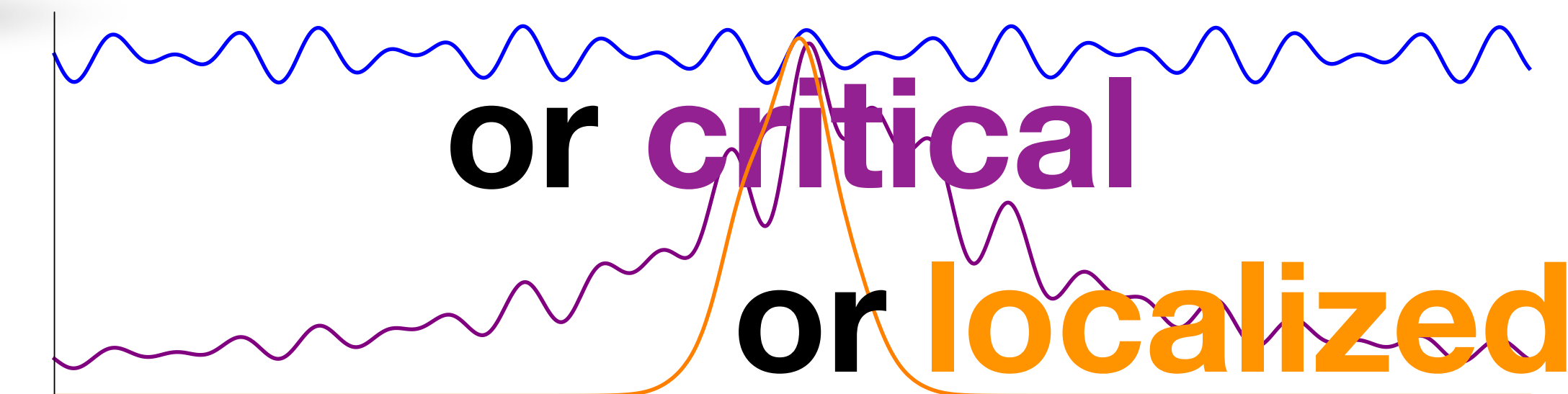
# probabilities for quantum particles in...

...crystals  
(periodic repetitions)



...quasicrystals  
(aperiodic)

**delocalized**  
or **critical**  
or **localized**

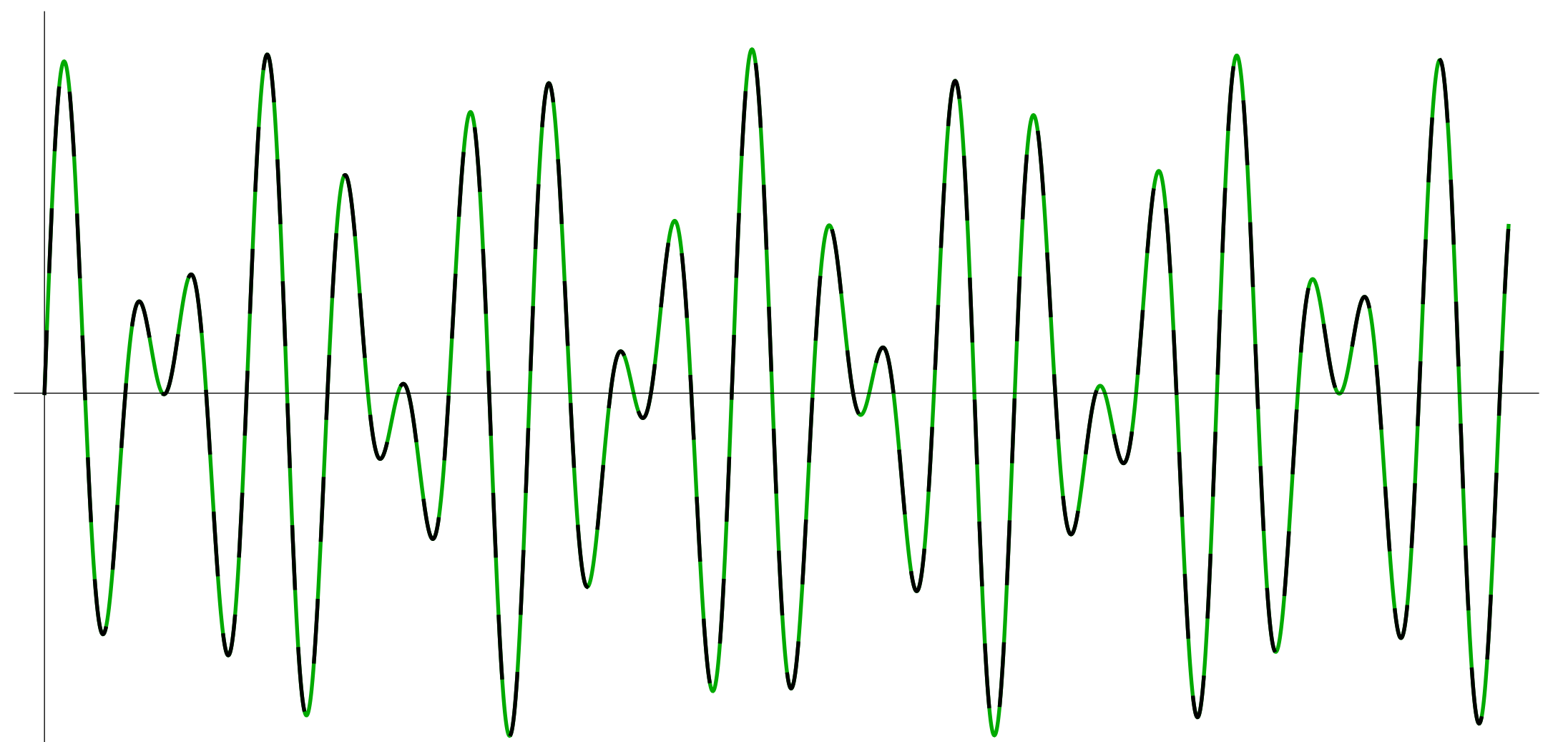
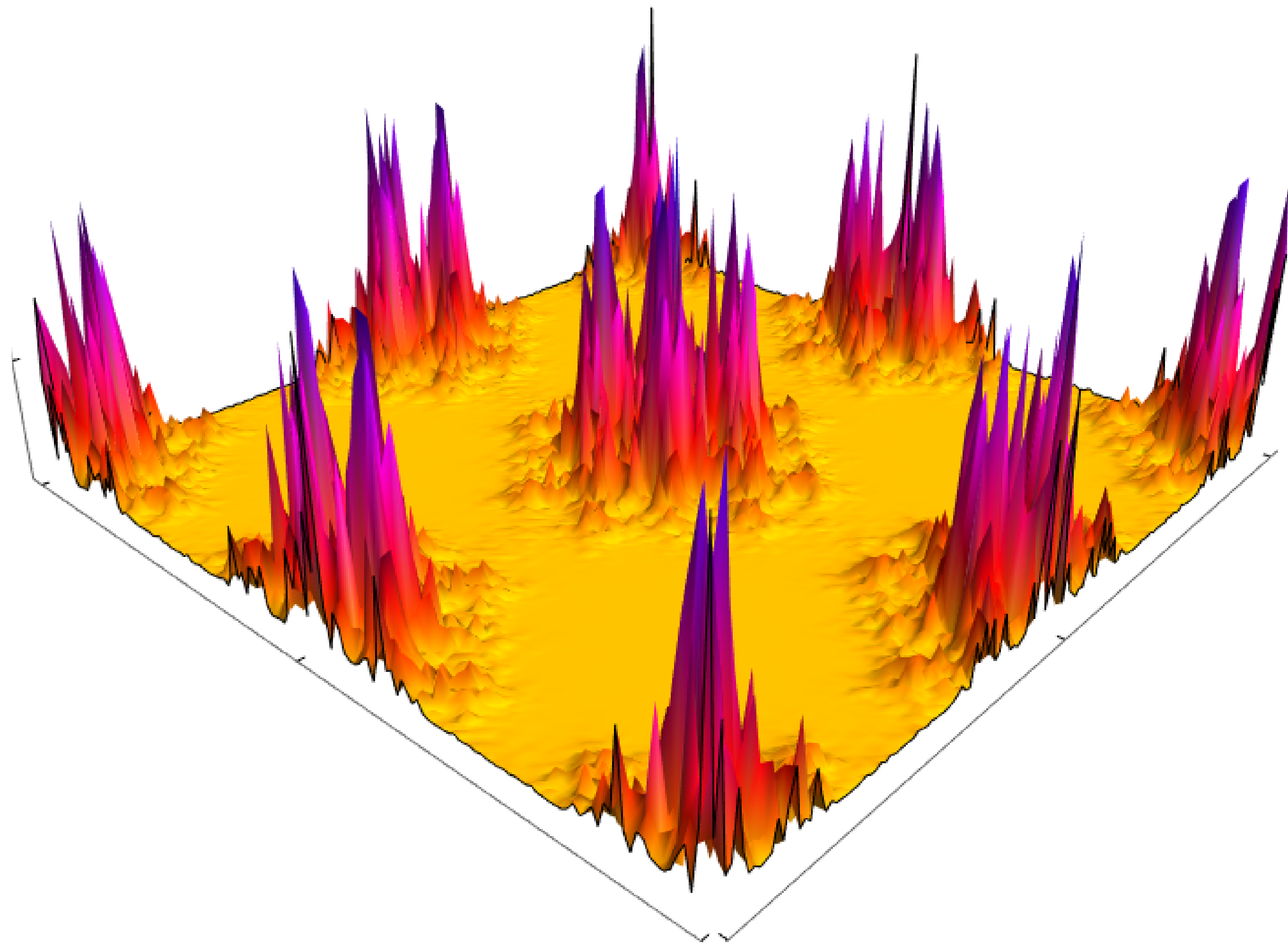




$|\Psi(x)| = |\Psi_0| e^{-\frac{x}{2a}}$   $\Psi_1$   $\hat{H}\Psi_a = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_a(x) = -\frac{\hbar^2}{2m} \frac{1}{4a^4} (x-x_0)^2 \Psi_a(x)$   
 $\sigma = 10^{-18} \text{ m}$   $\hat{H}\Psi_c = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_c = E_c \Psi_c$

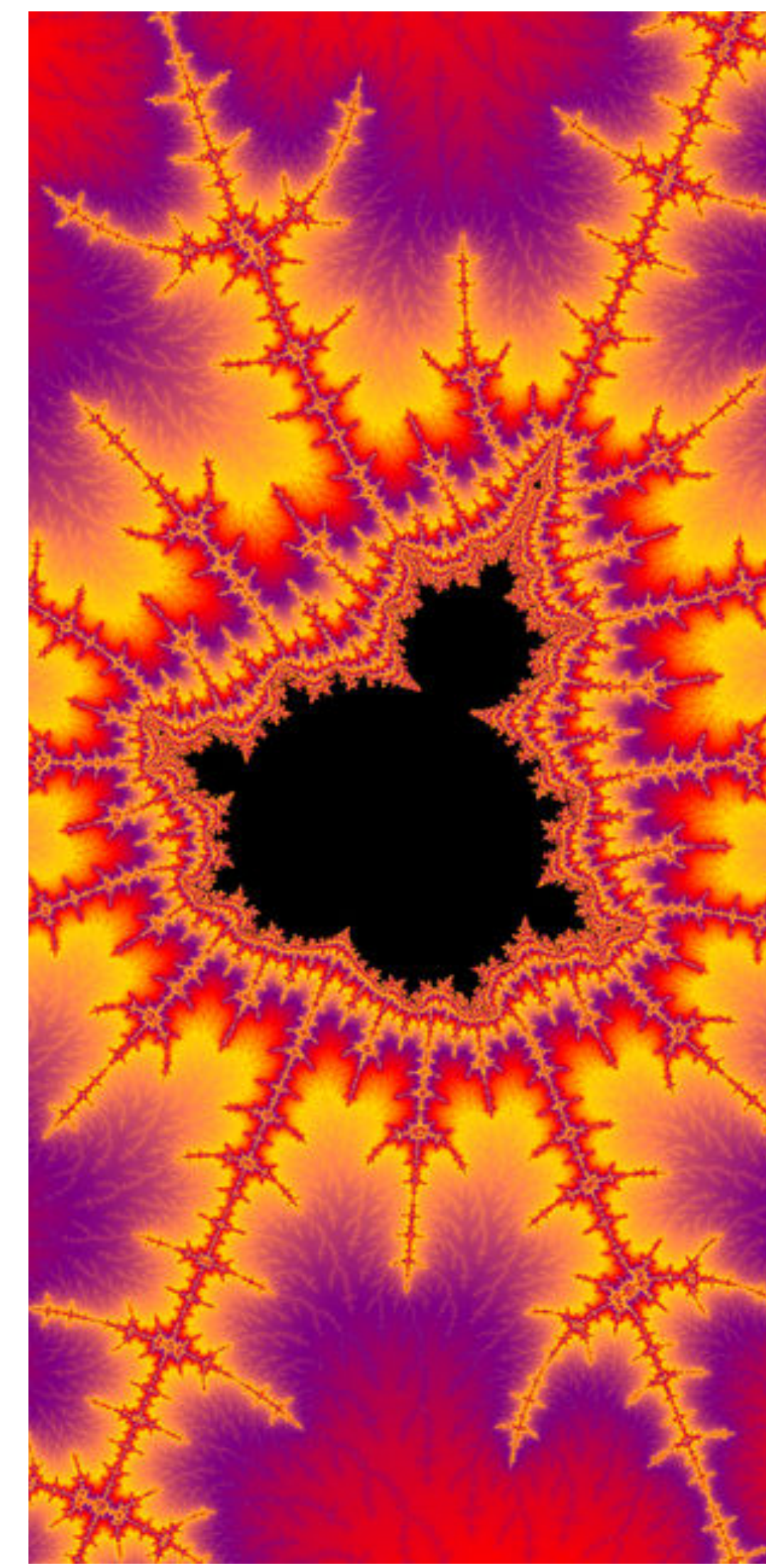
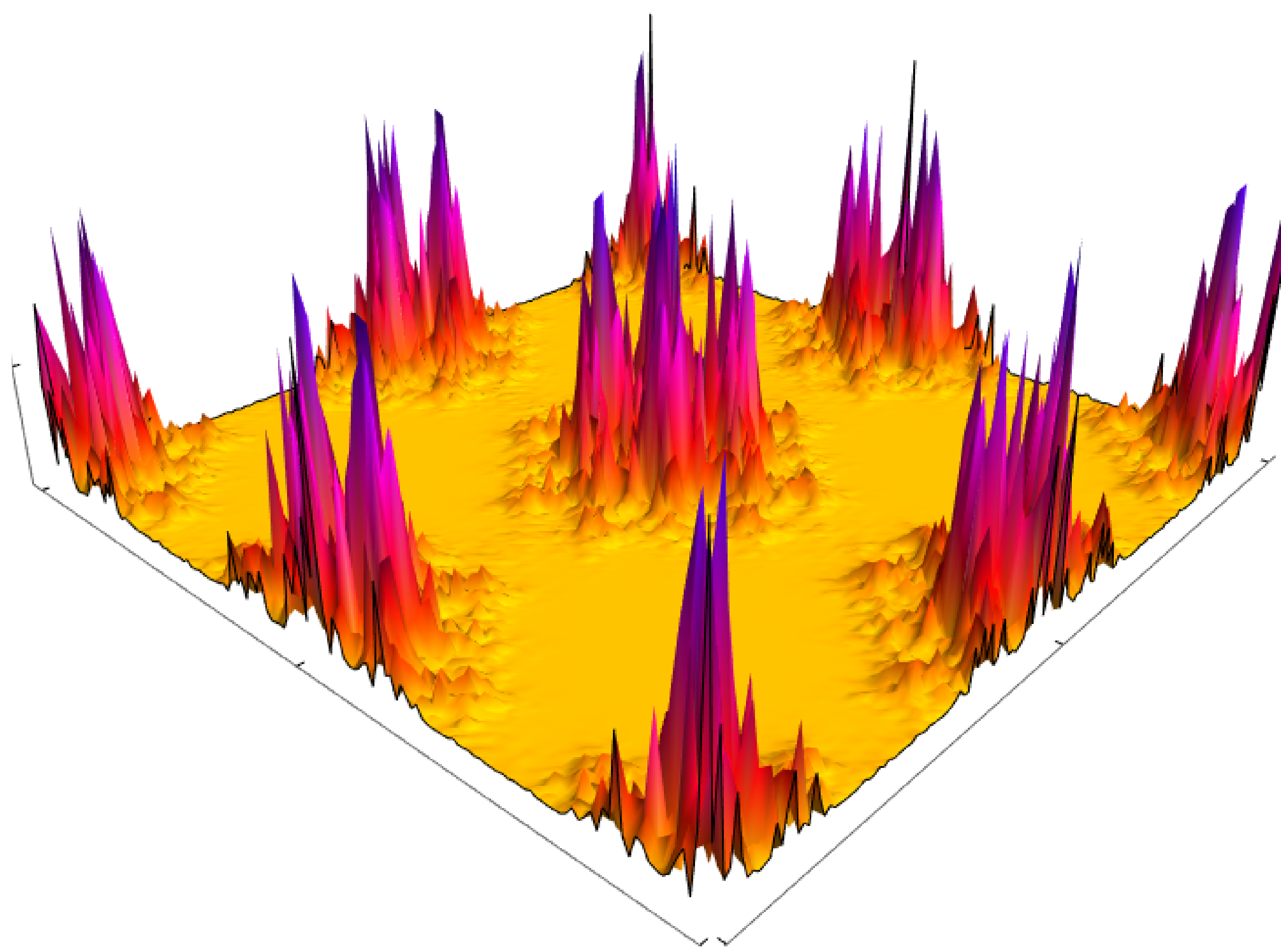
# probabilities for quantum particles in...

...quasicrystals  
(aperiodic)



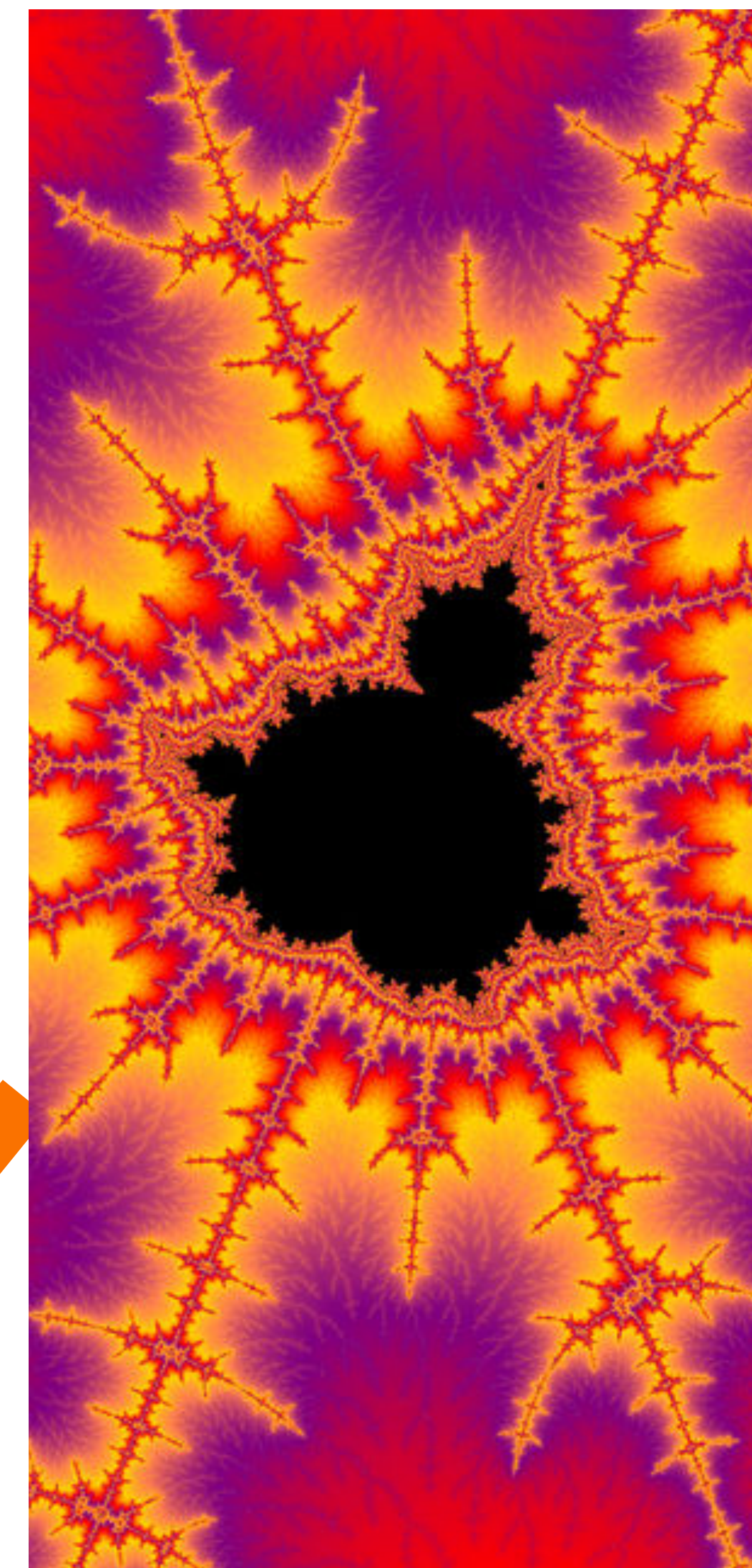
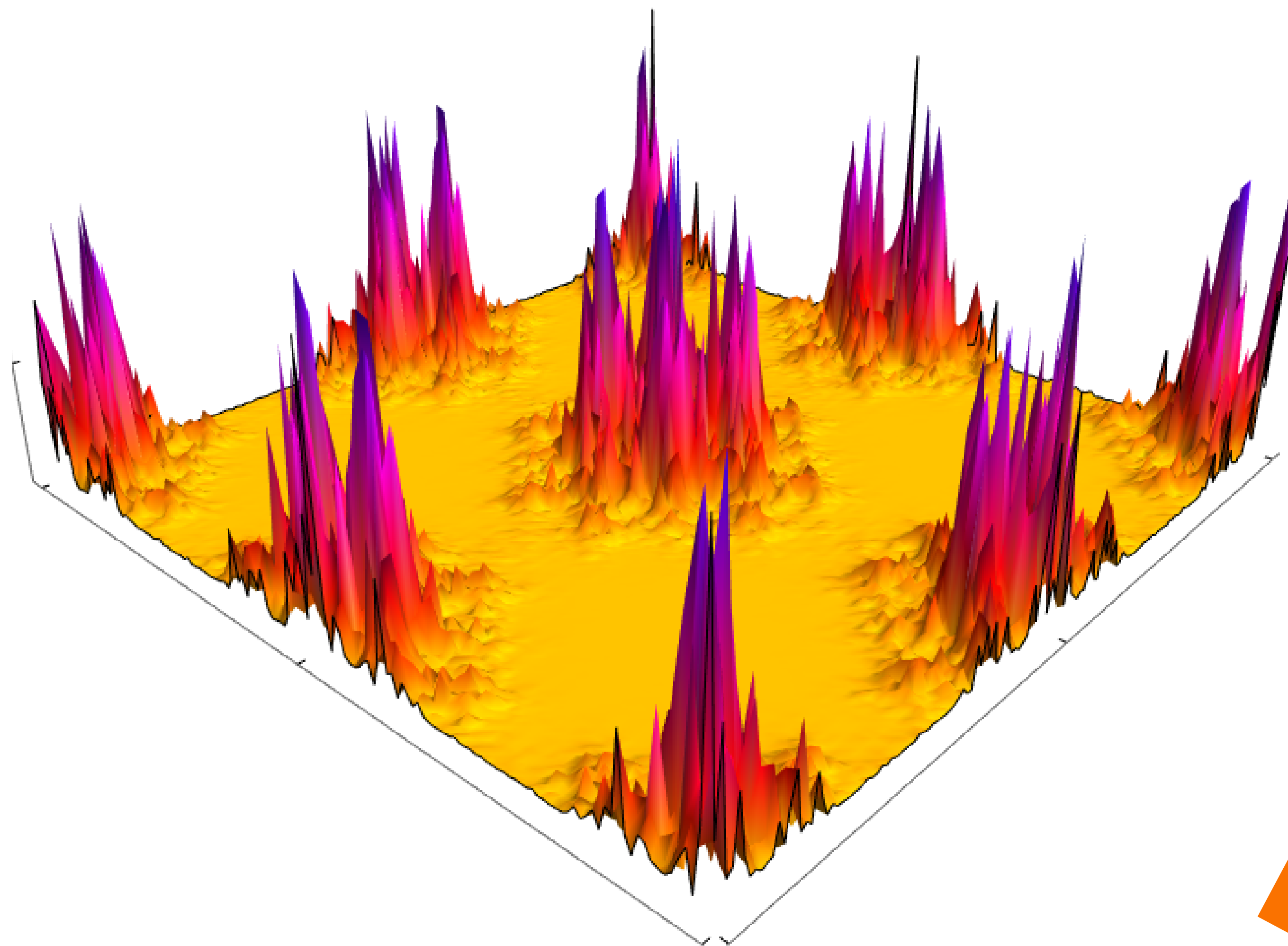


**probabilities for quantum particles in...**



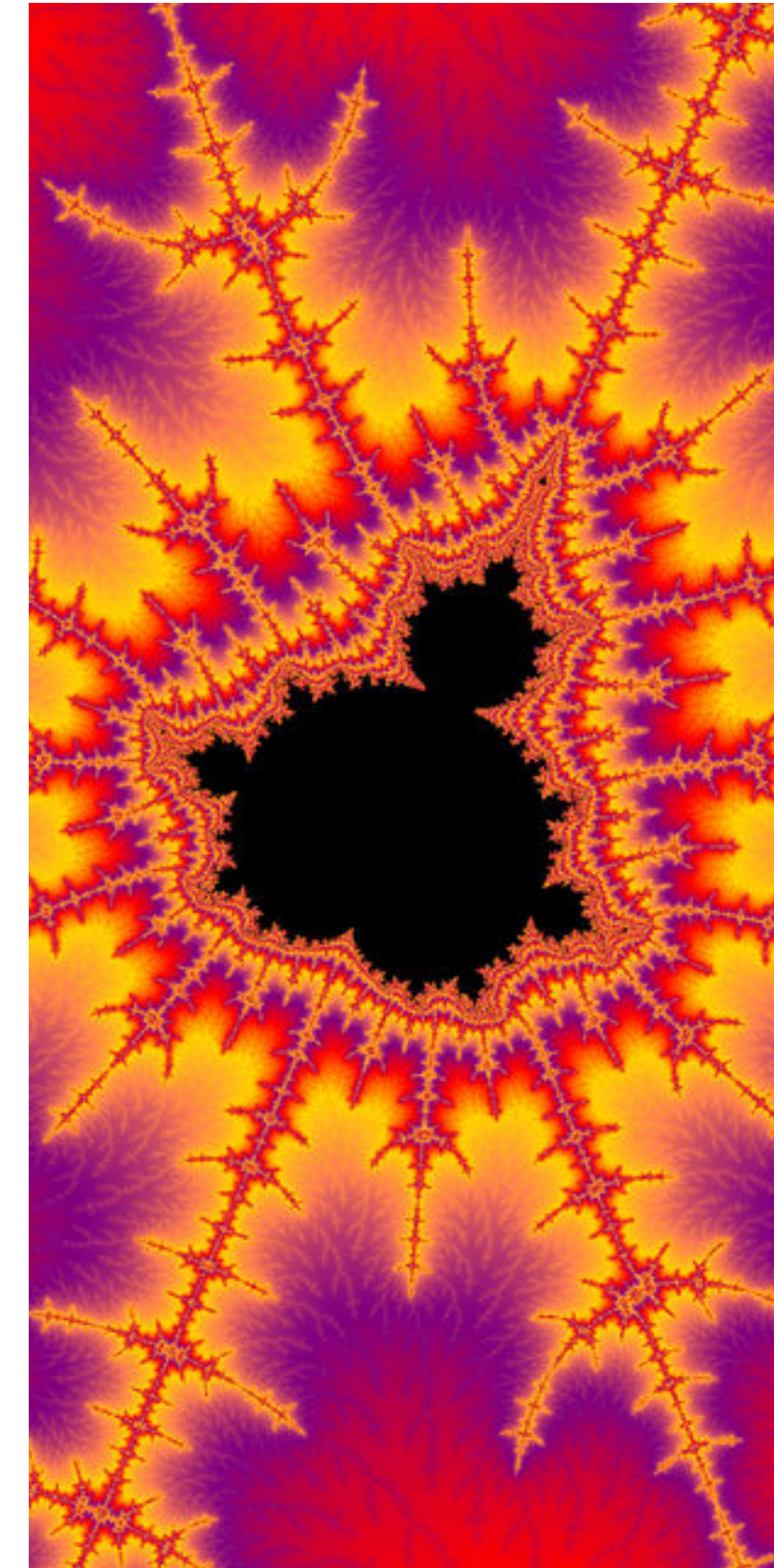
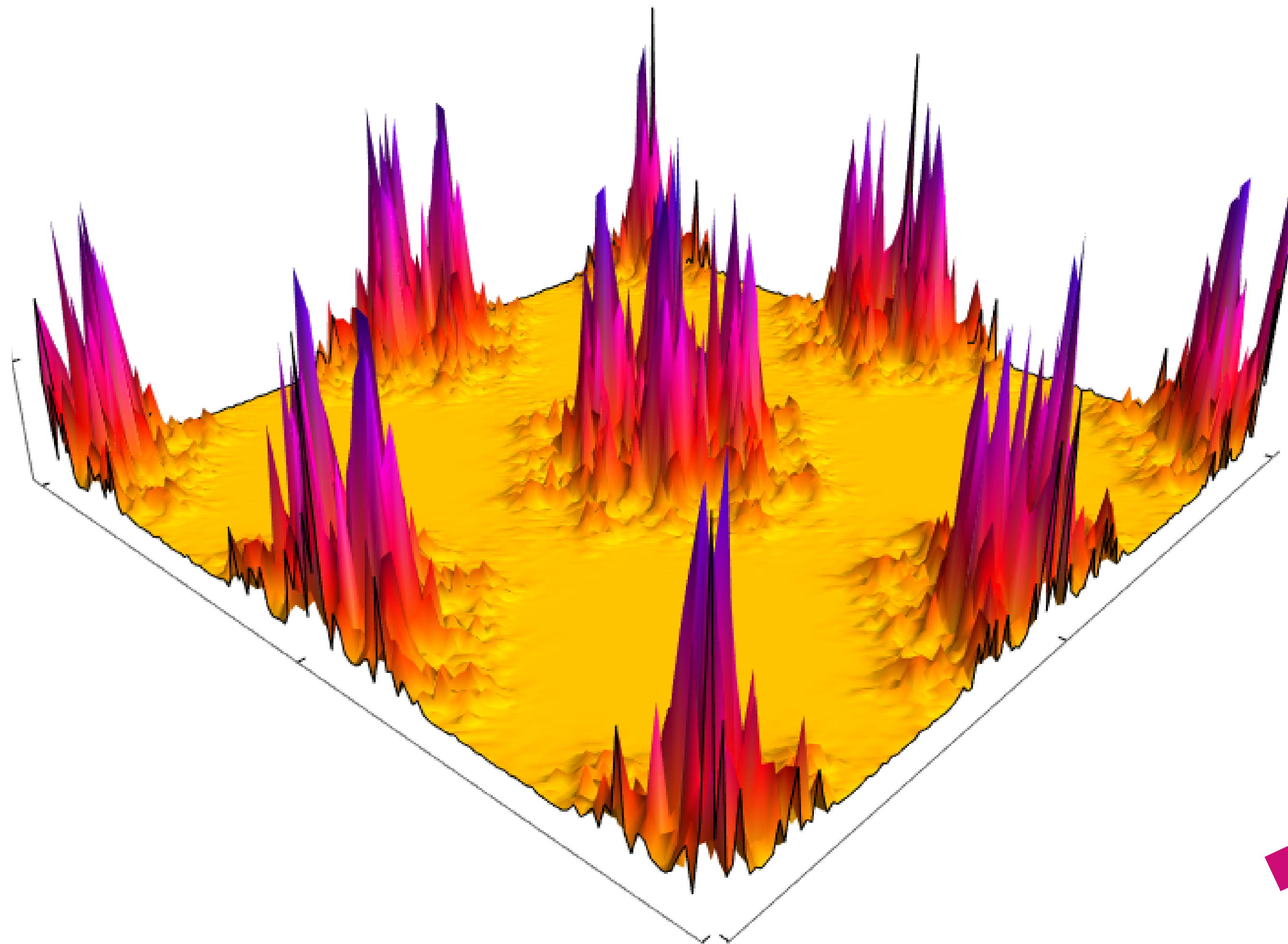


# Coastlines of dimension 1.25



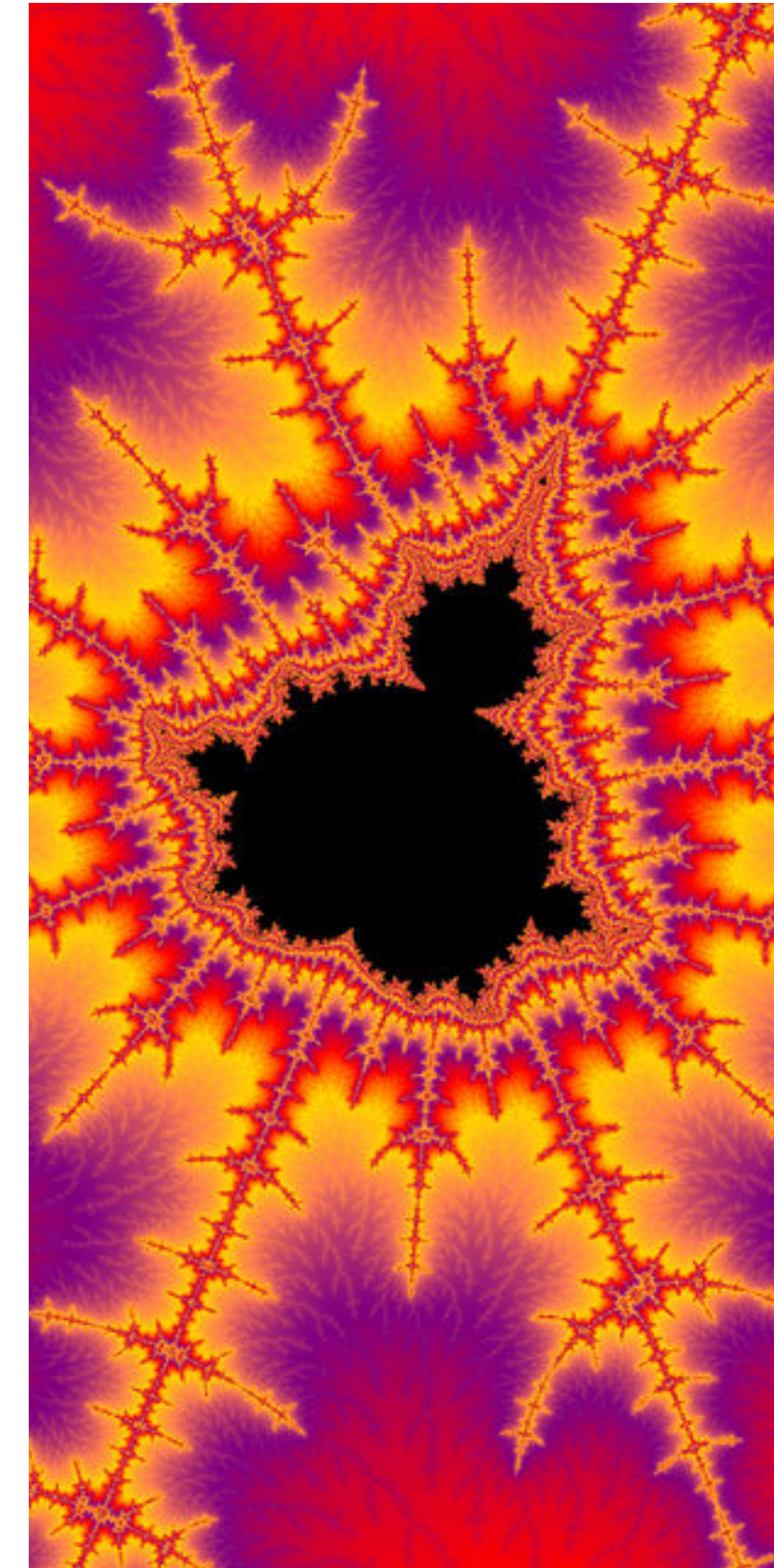
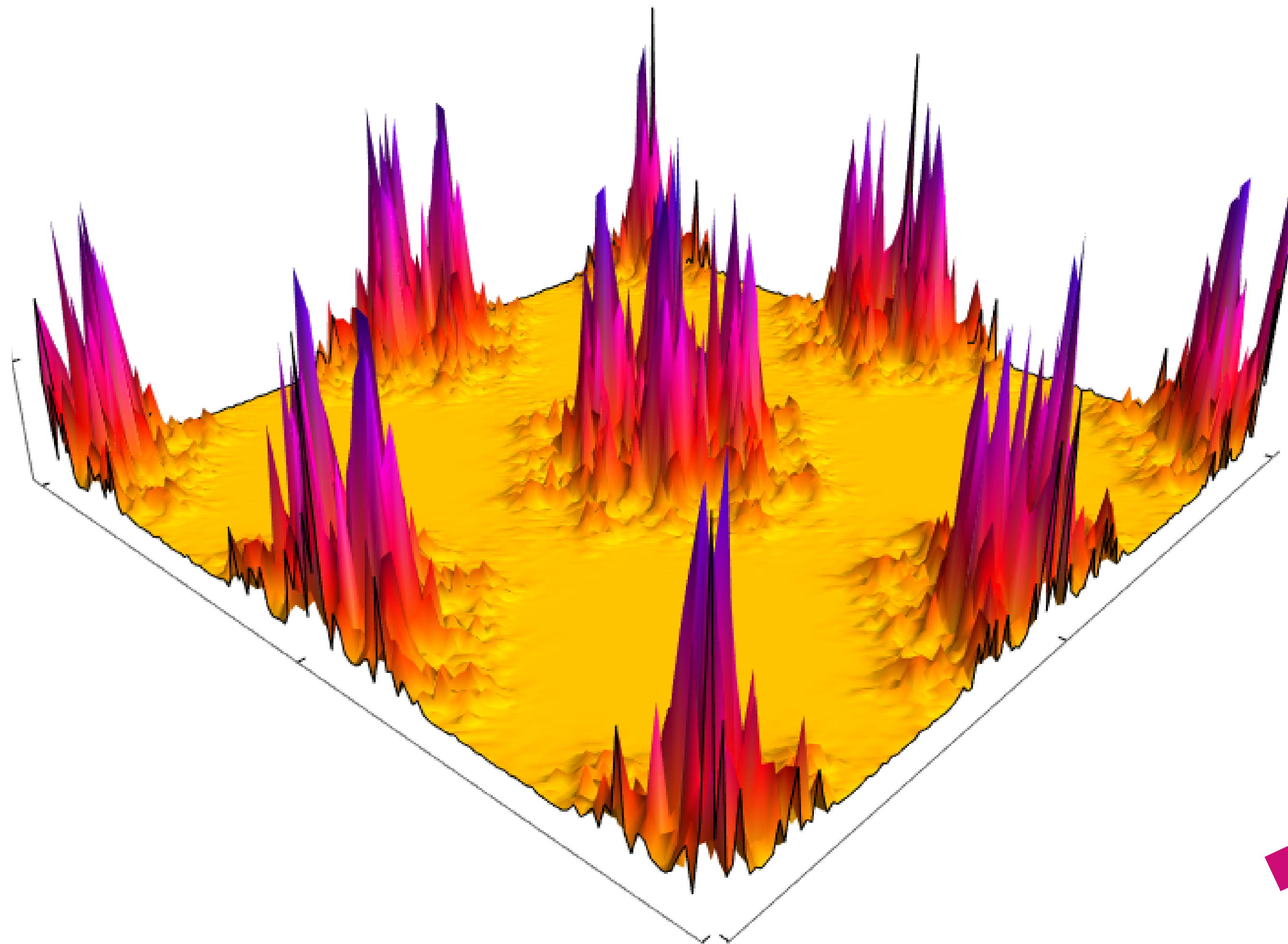


# Coastlines of dimension 1.25

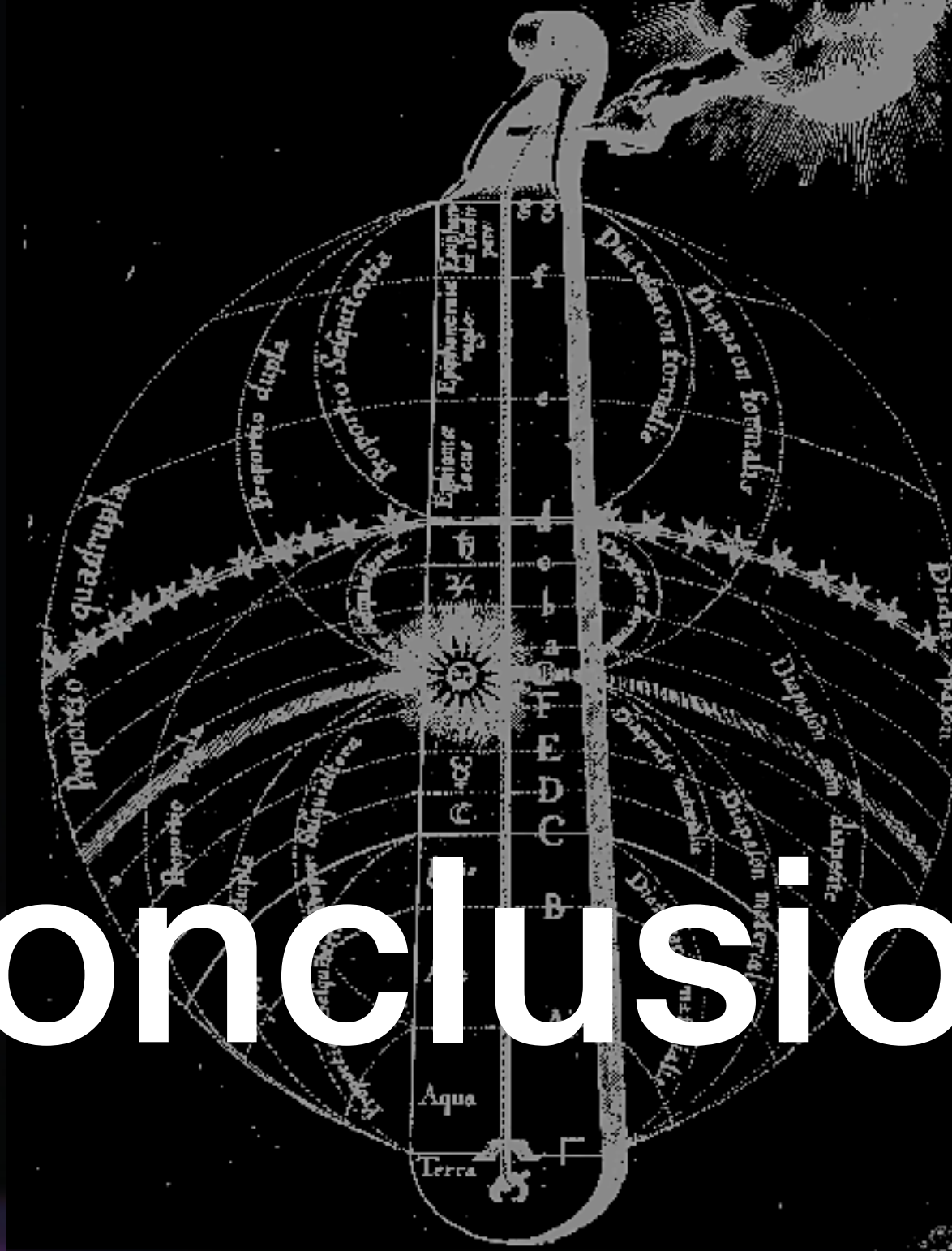




# Coastlines of dimension 1.25







Conclusion:

from

the perspective of quantum physics  
equal temperament isn't all that boring

Elio König-Tarasevich